Causal Set Cosmology

因果集合宇宙学

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In the absence of direct experimental/observational data and viable low-energy limits, theories of quantum gravity should look toward cosmology as their testing ground. Presented here is a summary of ideas rooted in causal set theory about some solutions to a few of the problems of cosmology.

在缺乏直接实验/观测数据以及可行低能极限的情况下，量子引力理论应当将宇宙学作为其检验场。本文总结了源自因果集理论、针对若干宇宙学问题的一些解决方案的相关思路。

Keywords

关键词

Causal set theory - Dark energy - Cosmological constant - The early universe

因果集合理论 - 暗能量 - 宇宙学常数 - 早期宇宙

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Introduction

引言

Our current understanding of the natural laws rests on two pillars of modern physics - quantum theory (QT) and general relativity (GR). Together these theories have been able to explain almost all of the observations, but it is generally accepted that the picture as a whole is not complete, that is, GR and QT are not compatible with each other. The two theories are very different and not just in their outlook but in their essence and also in their range of applicability. By outlook we mean that GR is deterministic in the classical sense where reality is unambiguous and the future is predictable, at least in principle [1, 2]. QT, on the other hand, is not deterministic in the classical sense in that the results of measurements/observations cannot be predicted beforehand even when we thoroughly know the initial state of the system under consideration [3,4]. Reality, in the sense of ”what actually exists and is fundamental” is not so clear.

我们目前对自然规律的认知建立在现代物理学的两大支柱之上——量子理论 (QT) 与广义相对论 (GR)。这两个理论加在一起几乎可以解释所有观测结果，但学界普遍认为这幅图景整体并不完整，也就是说广义相对论和量子理论彼此并不相容。二者不仅核心观点不同，本质和适用范围也存在极大差异。我们所说的核心观点差异指: 广义相对论是经典意义上的决定论，认为实在是明确的，未来至少在原理上是可预测的 [1,2]。而量子理论并不属于经典意义上的决定论: 即使我们完全掌握所研究系统的初始状态，测量/观测的结果也无法提前预测 [3,4]。而“什么实际存在、什么是根本”意义上的实在，就更不明确了。

This deterministic vs probabilistic outlook, although, a very important difference in its own right, is not the essence of the divide between the two theories, which comes from two main ideas of QT that are absent in GR as in any other classical theory, namely, ”interference” and ”non-locality.” In QT alternatives interfere as is very neatly demonstrated by the two (or more) slit experiment [4-6]. Similarly, GR like its predecessor classical theories is local in nature, where, say, the trajectory of a test particle is completely determined by the information in its immediate neighborhood. This is no longer true in QT - a fact that is quite clearly (and experimentally) demonstrated by the phenomenon of entanglement [7, 8] .

不过，这种决定论与概率观的分歧本身虽然非常重要，却并非两个理论分歧的核心；两个理论的核心分歧来自量子理论中两个不存在于广义相对论 (也不存在于其他任何经典理论) 的核心概念，即“干涉”和“非定域性”。在量子理论中，不同可能性会发生干涉，双 (多) 缝实验就很好地证明了这一点 [4-6]。与之类似，广义相对论和它之前的经典理论一样，本质上是定域性的: 例如，测试粒子的运动轨迹完全由其邻近区域的信息决定。这一点在量子理论中不再成立，纠缠现象 [7, 8] 就已经清晰地 (并且在实验上) 证明了这个事实。

An immediate question now arises; if the two theories are so different from each other, surely they can't both be true, and yet we have been developing both of them for the last one hundred or so years side by side. The answer to this question lies in the fact that the two have completely disparate domains of applicability and hence we could understand/explain all of our experiments/observations using only one of the two theories. Everything in the domain of "the large," for example, cosmology [9], gravitational waves [10], and black holes [11], could be made sense of using GR, whereas all microscopic calculations about things like atoms, nuclei, and subatomic particles could be performed solely with QT. So GR became the theory of "the large" and QT that of "the small." In other words, the two don't talk to each other, and hence there is no conflict whose resolution could lead to further development. In fact, in their respective domains, we haven't been able to find anything observationally wrong with either of them. Things on the quantum side got together into what is called the standard model of particle physics [12] about half a century ago, and it is considered by many to be an extremely successful theory with immense predictive and calculational power. GR hasn't been far behind with predictions like black holes and gravitational waves coming spectacularly true and the standard model of cosmology [9] being able to accommodate/explain all sorts of observations like the light element abundances [13,14], cosmic microwave background [15, 16], expanding universe, and the structure formation [17] in the latter stages of the evolution of the universe.

现在一个直接的问题应运而生: 如果这两种理论彼此差异如此之大，它们肯定不可能同时成立，然而在过去约一百年里，我们一直并行发展着这两种理论。这个问题的答案在于，二者的适用范围完全不同，因此我们仅用其中任意一种理论，就能理解或解释所有的实验与观测结果。“大尺度”领域中的所有事物，比如宇宙学 [9]、引力波 [10] 和黑洞 [11]，都可以用广义相对论 (GR) 解释；而所有关于原子、原子核、亚原子粒子这类微观对象的计算，都仅能通过量子理论 (QT) 完成。因此广义相对论成为了描述“大尺度”的理论，量子理论则成为描述“小尺度”的理论。换句话说，二者互不干涉，因此不存在需要解决的冲突来推动进一步发展。实际上，在各自的适用领域中，我们从未在观测中发现二者存在任何错误。大约半个世纪前，量子领域的研究成果整合为粒子物理标准模型 [12]，许多学者认为这是一个极为成功的理论，拥有强大的预测能力和计算能力。广义相对论也毫不逊色: 黑洞、引力波等预言都出色地得到验证，宇宙学标准模型 [9] 能够容纳或解释各类观测结果，例如轻元素丰度 [13,14]、宇宙微波背景 [15, 16]、宇宙膨胀，以及宇宙演化后期的结构形成 [17]。

We need a way out of this dilemma of how to reconcile two very different but equally successful theories - a problem that goes by the generic name of quantum gravity. The discussion above suggests an obvious solution. We need to perform experiments or look for observations where both of the theories are important and neither could be ignored, which brings us to the seemingly impossible task of finding a phenomenon that belongs to both in the large and in the small at the same time. Let us take a closer look at the separation between the domains of the two theories in order to come to better grips with what we are suggesting.

我们需要找到方法走出这个“如何调和两个差异巨大却又同样成功的理论”的困境，这个问题一般被统称为量子引力。上文的讨论指向了一个明确的方向: 我们需要开展实验，或是寻找观测中同时涉及两个理论、二者都不可忽略的情况，这就带来了一个看似不可能的任务: 寻找一种同时兼具“大尺度”和“小尺度”属性的现象。我们来更仔细地梳理一下两个理论的领域划分，从而更好地理解我们提出的方向。

It is true that ”the size” of a phenomenon decides which part of physics will be used to study it, but it really is a reflection of whether we are interested in ”the instantaneous” or the ”the average” part of the physical behavior. QT is important when we choose the size and/or time and/or energy involved to be so small that only a few degrees of freedom of the system are excited, which usually means ”the small.” But if we have a relatively large system which is near its ground state and we excite only a small number of degrees of freedom, we can observe quantum behavior in the large as well. An example would be a Fermi gas [18] or a Bose-Einstein condensate [19]. If we follow this lead into quantum gravity, we need to look for a gravitational phenomenon with only a small number of degrees of freedom. A Planck-size black hole, that is, a Planck-mass black hole whose event horizon has an area of a square Planck-length, the gravitational waves generated by the merger of two Planck-size black holes, and a universe that is Planckian in either ”spatial size” or age or both (sometimes called the very early universe) are such examples. Unfortunately, the length scales involved are so small (or equivalently the energy scales needed are so large) that we cannot achieve them with our current technologies in any of our laboratories. In fact, we cannot hope to achieve these length scales/energies with the current technologies in any ”reasonable way” in any ”foreseeable future.” This has been the generic problem with any candidate solution of quantum gravity, such as String theory [27], loop quantum gravity [28], causal sets [29], and causal dynamical triangulation [30] to name a few examples. These theories are formulated at such high-energy scales that it is impossible to test them directly. In the absence of direct testing, the second best alternative is to develop the low-energy limits of the candidate solutions. This is a work in progress. Until we develop technologies and experiments that can do the direct testing, or develop proper low-energy limits, cosmology and the very early universe seem to be the most promising ways of making progress. This is because there are established cosmological observations that we theoretically understand very little of, and for some of them, their origin suggests a deep connection with QG. We mention some of them below.

确实，一个现象的“规模”决定了将用物理学的哪一部分来研究它，但这实际上反映了我们是对物理行为的“瞬时”部分还是“平均”部分感兴趣。当我们选择所涉及的规模、时间和/或能量小到系统只有几个自由度被激发时，量子理论 (QT) 就很重要，这通常意味着“小尺度”。但如果我们有一个相对较大的系统且接近其基态，并且只激发少量自由度，我们也能在大尺度上观察到量子行为。例如费米气体 [18] 或玻色 - 爱因斯坦凝聚体 [19]。如果我们沿着这个思路研究量子引力，我们需要寻找只有少量自由度的引力现象。普朗克尺度的黑洞，即视界面积为一个普朗克长度平方的普朗克质量黑洞、两个普朗克尺度黑洞合并产生的引力波，以及在“空间规模”、年龄或两者上都具有普朗克尺度的宇宙 (有时称为极早期宇宙) 就是这样的例子。不幸的是，所涉及的长度尺度非常小 (或者等效地，所需的能量尺度非常大)，以至于我们目前的任何实验室技术都无法达到这些条件。事实上，在“可预见的未来”，我们无法以任何“合理的方式”用现有技术达到这些长度尺度/能量。这一直是量子引力任何候选解决方案的普遍问题，比如弦理论 [27]、圈量子引力 [28]、因果集 [29] 和因果动态三角剖分 [30] 等。这些理论是在如此高的能量尺度上构建的，以至于无法直接对其进行测试。在无法进行直接测试的情况下，次优的选择是研究候选解决方案的低能极限。这是一项正在进行的工作。在我们开发出能够进行直接测试的技术和实验，或者开发出合适的低能极限之前，宇宙学和极早期宇宙似乎是取得进展最有希望的途径。这是因为有一些已确立的宇宙学观测现象，我们在理论上对其了解甚少，而且其中一些现象的起源表明它们与量子引力有深刻的联系。我们在下面会提及其中一些现象。

The Puzzle of the Initial Conditions

初始条件谜题

Planckian energy scales, which correspond to a temperature of around 10^{32} K (around 2×10^{19} J in a Planck volume, that is, around 10^{-105} m^3) cannot be generated in our laboratories, but the early universe, which is the hottest place that we know of, may have gone through a phase which involved such energy scales. In fact, as mentioned earlier, it is one of the prime places to look for a merger of QT and GR and, hence, is worth mentioning a bit more.

普朗克能标对应的温度约为 10^{32} K (在普朗克体积中约为 2×10^{19} J，即约为 10^{-105} m^3)，无法在我们的实验室中产生，但早期宇宙作为我们已知温度最高的存在，可能经历过包含这类能标的阶段。事实上，正如前文所述，它是寻找量子理论与广义相对论融合的核心场所之一，因此值得多做一些介绍。

The universe seems to be expanding isotropically and homogeneously [9, 20- 23]. This means that the space (on the cosmological scale) is expanding such that if the distance between any two pairs of points/locations is the same, then the rate at which they recede from each other is also the same, and this rate does not depend on the relative directions of these pairs with respect to us (isotropy) or their location (homogeneity). The current rate of this expansion - a number known as the Hubble constant (H) - is about 70 km/s/Mpc [24, 25] , which means that any two locations separated by a distance of 1Mpc are receding from each other at a speed of 70 km/s . Two side points are worth noting. Firstly, we expect the cosmological principle (and hence this number) to hold on a distance of around 200-300 Mpc, and secondly, since the rate of expansion changes with time, H despite being called the Hubble constant is not really a constant in time. The gravitational effect of matter (and energy) tries to slow down the rate of this expansion. On the other hand, there are possible contributions to the energy contents of the universe that have an opposite effect. Depending on these contents of the universe and their properties, the universe may or may not be able to stop this expansion [9]. According to our current understanding, the universe at the present epoch is dominated by two components - matter and dark energy [9, 26] . Matter, which is around thirty percent of the total energy density, tries to slow down the expansion rate of the universe. The rest is found as a negative pressure perfect fluid that has the opposite effect and tries to accelerate the expansion rate.

宇宙似乎在各向同性且均匀地膨胀 [9, 20-23]。这意味着在宇宙学尺度上，空间膨胀满足: 若任意两对点/位置之间的距离相等，那么它们相互退行的速率也相等，且该速率不依赖于这些点对相对于我们的方向 (各向同性)，也不依赖于它们的位置 (均匀性)。当前的膨胀速率即哈勃常数 (H) 约为 70 km/s/Mpc [24, 25] ，这意味着相距 1Mpc 的任意两个位置会以 70 km/s 的速度相互退行。有两点需要额外说明: 第一，我们认为宇宙学原理 (以及该数值) 适用于约 200-300 百万秒差距的尺度；第二，由于膨胀速率随时间变化，H 尽管被称为哈勃常数，它实际上并非不随时间变化的常数。物质 (和能量) 的引力效应会试图减缓膨胀速率，另一方面，宇宙的能量组分中可能存在起到相反作用的贡献。根据宇宙的这些组分及其性质，膨胀未必无法被停止 [9]。就我们目前的认知而言，当前宇宙由两种组分主导——物质和暗能量 [9, 26] 。物质约占总能量密度的百分之三十，它会试图减缓宇宙的膨胀速率；剩余部分是具有负压强的理想流体，起到相反的作用，会试图加速膨胀速率。

This expansion is causing the universe to cool down as can be measured by the temperature of the cosmic microwave background (CMB). If we reverse the direction of time, the expansion becomes an isotropic and homogeneous contraction and the universe starts to heat up. The farther back we go in time, the smaller and consequently the hotter is the universe. This conclusion depends heavily on the trust we put in GR, and if we take GR to be valid for all length and time scales, we end up with an infinitely hot universe - the initial singularity. Of course, we do not trust GR for arbitrarily small time and length scales, and hence we are not certain if there was an initial singularity in the classical sense, but going back in time if the universe does become Planckian in size, it is precisely the very early universe we alluded to earlier.

这种膨胀会导致宇宙降温，这一点可以通过宇宙微波背景 (CMB) 的温度观测得到。如果我们将时间倒流，膨胀就会变成各向同性、均匀的收缩，宇宙开始升温。我们回溯的时间越早，宇宙的体积越小，温度也就越高。这个结论高度依赖于我们对广义相对论的信任，如果认为广义相对论在所有长度和时间尺度下都成立，我们最终会得到一个温度无限高的宇宙——初始奇点。当然，我们并不认为广义相对论适用于任意小的时间和长度尺度, 因此我们不确定经典意义上的初始奇点是否真的存在；但如果回溯时间后宇宙确实收缩到普朗克尺度，那这正是我们之前提到的极早期宇宙。

The proper physics to study the very early universe has to be QG by the logic developed earlier. Of course, we have not been able to observe this epoch directly, but as the universe comes out of this very early phase, it has some puzzling properties called the initial conditions, which determine the subsequent evolution of the universe. If the initial moments of the universe, say, until the universe is a Planck time old, are governed by QG, then we should be able to explain or, at the very least, naturally accommodate this set of initial conditions within a framework based on our candidate QG theory. It should be mentioned here that there have been other attempts to explain this set of initial conditions that start off ”the post-Planckian era” called the early universe. Most notable among these is the inflationary hypothesis [32]. We will talk about two of these initial conditions in particular as there are indications that they arise naturally in models having roots in causal set theory - a promising candidate for the solution of QG problem and to be discussed in the next section.

根据前文的逻辑，研究极早期宇宙的正确物理理论必然是量子引力。我们当然无法直接观测这个时期，但当宇宙走出这个极早期阶段后，它会带有一些令人困惑的性质，也就是初始条件，这些初始条件决定了宇宙后续的演化。如果宇宙的初始时刻 (比如直到宇宙年龄达到普朗克时间之前) 由量子引力主导，那么我们应当能够在候选量子引力理论的框架下解释，或者至少自然容纳这一组初始条件。需要说明的是，目前已经有其他从“后普朗克时代”(即早期宇宙) 出发解释这组初始条件的研究，其中最受关注的是暴涨假说 [32]。我们接下来会具体讨论两个初始条件，因为有迹象表明，它们在源自因果集理论的模型中会自然出现——因果集理论是量子引力问题很有前景的候选理论，我们会在下一节讨论它。

The first of these puzzling initial conditions is the concern that as the universe enters into its post-Planckian epoch, it is surprisingly very large. In other words, we expect the universe to be spatially of Planckian size when it is a Planck time old, but it turns out to be something like 30 orders of magnitude larger than that [31]. This means that when the universe was only one Planck time old it was composed of around 10^{90} causally disconnected Planck size regions (in standard cosmology). These regions are causally disconnected because light/information can only travel one Planck length in one Planck time. It is worth mentioning here that in inflationary scenario this problem is tackled by assuming that a part of the very early universe undergoes exponential expansion caused by a very special scalar field, called the inflaton, and this super expansion produces the observed early universe.

这些令人困惑的初始条件中，第一个问题是: 当宇宙进入后普朗克纪元时，它的尺寸大得异乎寻常。换句话说，我们本预期宇宙在普朗克时间年龄时，空间尺寸应为普朗克尺度，但实际结果却比这大了约 30 个数量级 [31]。这意味着，当宇宙才诞生一个普朗克时间时，它就由大约 10^{90} 个因果不相连的普朗克尺寸区域构成 (标准宇宙论中)。这些区域因果不相连，是因为光/信息在一个普朗克时间内只能传播一个普朗克长度。值得一提的是，暴胀场景通过假设极早期宇宙的一部分发生指数膨胀解决了这个问题，该膨胀由一种非常特殊的标量场 (即暴胀子) 驱动，这种超级膨胀产生了我们观测到的早期宇宙。

This ”absurdly large” observed size of the early universe is not only a puzzle in itself but brings us to our second puzzle as well. Why a universe that is so causally disconnected happens to be so homogeneous (and isotropic)? If we look at two diametrically opposite points in the sky, the light from the farthest away points in these two directions have just reached us, and hence the two regions have never been in causal let alone thermal contact. Still the two regions have the same temperature. This is like two alien species from different galaxies, who have still not made contact with each other, speaking the same language. As mentioned earlier, we will discuss later that there are reasons to believe that causal set theory might have something to say about these puzzles.

早期宇宙这种“大得荒谬”的观测尺寸本身就已是一个谜题，还引出了我们的第二个谜题。为什么一个因果区如此割裂的宇宙，竟会如此均匀 (且各向同性)? 如果我们观察天空中两个完全相反的方向，这两个方向最远点发出的光才刚刚到达地球，因此这两个区域从未产生过因果接触，更不用说热接触了。但这两个区域的温度仍然完全相同。这就好比来自不同星系、从未接触过对方的两个外星物种，说着同一种语言。正如前文所述，我们在后文会讨论，有理由认为因果集理论或许能为这些谜题给出解答。

Strange Cosmological Phenomena

奇异宇宙学现象

There are some very well-established natural phenomena that have defied explanation using contemporary physics. Examples include but are not limited to the acceleration in the expansion rate of the universe and the cosmic rays. In the absence of direct data to calibrate our QG theories against, we can build confidence in our candidate solutions to QG by saying something about these unexplained phenomena. In the following, we will focus on the dark energy with the intention of talking about a solution later in this capacity.

目前存在一些已得到充分确认、却无法用当代物理学解释的自然现象。这类现象包括但不限于宇宙膨胀速率的加速，以及宇宙射线。在没有直接数据可用于校准我们的量子引力理论的情况下，我们可以通过对这些未解现象给出解释，来提升量子引力候选理论的可信度。下文我们将聚焦暗能量，以便后续在此框架下讨论一套解决方案。

In cosmology, matter is everything that dilutes as number density, that is, whose energy density depends just on the number of particles in a given volume. This includes both baryonic and non-bryonic matter [9]. As the volume (of a region) changes from V_1 to V_2 , the density changes from ρ_1 to ρ_2 such that $V_1\rho_1 = V_2\rho_2$. In an isotropic and homogeneous universe, the expansion is completely described by the scale factor, a, which can be thought of as the ratio of distance between two galaxies (or any two points for that matter), say, when they first formed and now. Since $V_1/V_2 = a_1^3/a_2^3$, we have $\rho_{\text{matter}} \sim a^{-3}$. On the other hand, radiation is everything that dilutes as a^{-4} [31].

在宇宙学中，物质是所有随体积膨胀稀释的存在，即它的能量密度仅取决于给定体积内的粒子数。这包括重子物质与非重子物质 [9]。当一个区域的体积从 V_1 变为 V_2 时，密度从 ρ_1 变为 ρ_2 ，满足 $V_1\rho_1 = V_2\rho_2$ 。在各向同性的均匀宇宙中，膨胀完全由尺度因子 a 描述，尺度因子可理解为两个星系 (或任意两个点) 在形成之初与现在的距离之比。由于 $V_1/V_2 = a_1^3/a_2^3$ ，我们得到 $\rho_{\text{matter}} \sim a^{-3}$ 。另一方面，辐射是所有按 a^{-4} 规律稀释的存在 [31]。

It has now been generally accepted that contrary to all earlier expectations, our universe is accelerating in its expansion [33-37]. As mentioned earlier, the rate of this expansion depends on the contents of the universe. We directly observe that the universe contains ordinary matter as is present in the visible part of the galaxies and the intergalactic matter. The other directly observable component, which was dominant in the early universe but is negligible now, is radiation. Since both of these components try to slow down the rate of expansion, it was generally expected that the universe should be decelerating. Two research groups led by A. Riess [33] and S. Perlmutter [36] tried to measure this time dependence of the expansion rate of the universe (i.e., the time dependence of the Hubble parameter) using SN Ia as standard candles [9]. To everyone's surprise, they both found out that the universe is not slowing down in its expansion but is instead accelerating (also see [38]). Also the CMB is not completely uniform but there are fluctuations of the order of 10^{-5} . The strength of these fluctuations on different length scales also carries information about the total energy density of the universe [39-41] irrespective of which component it is coming from. When the CMB fluctuations data (along with other considerations such as structure formation) is superimposed with the SN Ia result, two things become clear. Firstly, the universe does not just have matter and radiation, but there is a component of the universe whose energy density does not change with the expansion of the universe and behaves like the vacuum energy. This component, the so-called dark energy [26,42,43], is not only present but dominates the energy content of the universe being about 70 percent of it. Secondly, the visible or luminous matter, that is, the ”ordinary” matter made up of atoms that interacts with photons, is just 20 percent of the total matter content of the universe. The other 80 percent is the kind of matter that doesn't interact electromagnetically and hence is termed the dark matter [9]. These results were then strengthened by other arguments like the evolution of the structure on the cosmological scale and requirements on the minimum age of the universe. There have been several suggestions to explain the nature of the dark energy [44-60], but it is generally accepted that they suffer from naturalness and/or fine-tuning problems. Some are in conflict with other observations.

现在人们普遍认可，宇宙的膨胀打破了早期所有预期，正在加速 [33-37]。如前文所述，膨胀速率取决于宇宙的内容物。我们直接观测到，宇宙中存在普通物质，包括可见星系内部和星系间的物质。另一类可直接观测的组分是辐射，它在早期宇宙占据主导，如今占比可以忽略。由于这两类组分都会减缓膨胀速率，人们原本普遍认为宇宙膨胀应当在减速。A. 里斯 [33] 和 S. 珀尔马特 [36] 分别领导的两个研究小组，利用 Ia 型超新星作为标准烛光测量宇宙膨胀速率的时间依赖性 (即哈勃参数的时间依赖性)[9]。出乎所有人意料，两个团队都发现宇宙膨胀没有减速，反而在加速 (另见 [38])。此外，宇宙微波背景并非完全均匀，存在幅度约为 10^{-5} 的涨落。不同尺度涨落的强度也携带了宇宙总能量密度的信息 [39-41]，和涨落来自哪类组分无关。将宇宙微波背景涨落数据 (结合结构形成等其他考量) 与 Ia 型超新星的结果结合后，两点结论清晰浮现。首先，宇宙不只有物质和辐射，还存在一类组分，它的能量密度不随宇宙膨胀改变，性质与真空能一致。这类组分就是所谓的暗能量 [26,42,43]，它不仅存在，还占据了宇宙总能量的约 70%，主导了宇宙的能量组成。其次，可见 (发光) 物质，即由原子构成、能与光子相互作用的“普通”物质，仅占宇宙总物质的 20%。剩下的 80% 是不发生电磁相互作用的物质，因此被称为暗物质 [9]。这些结果随后得到了其他论证的支持，比如宇宙学尺度结构演化、宇宙最小年龄要求等。人们已经提出了多种解释暗能量本质的方案 [44-60]，但普遍认为这些方案都存在自然性和/或精细调谐问题，部分方案还和其他观测结果冲突。

We don't have much to say about the dark matter, but the most natural explanation for the existence of the dark energy is to think of it as the cosmological constant [61], Λ , first introduced by Albert Einstein into his famous Einstein's field equations of GR [2] to make the universe static and later discarded when the universe was shown to be expanding. It is possible to think of Λ as the energy that each unit 4-volume of spacetime has just by virtue of its existence. It is the same everywhere and hence it doesn't dilute as the universe expands. As each new unit of volume comes into being, it brings the same amount of energy with it and hence its energy density doesn't change.

我们目前对暗物质没有太多可说的, 但解释暗能量存在最自然的思路就是将其视为宇宙学常数 [61]，Λ ，它最初由阿尔伯特·爱因斯坦引入著名的广义相对论场方程 [2]，目的是得到一个静态宇宙，后来在发现宇宙膨胀后被爱因斯坦放弃。我们可以将 Λ 理解为每单位四维时空仅凭借自身存在就拥有的能量。它处处均匀，因此不会随宇宙膨胀被稀释: 每产生一个新的单位体积，就自带等量的能量，因此它的能量密度保持不变。

It seems like it is possible to think of the dark energy as the cosmological constant but close inspection reveals a few problems. We expect the value of the cosmological constant to be of order unity in natural units, i.e., each Planckian 4-volume should have Planckian energy. The observed/best-fit value of the energy density is around a 120 of magnitude smaller than this theoretical expectation, a mismatch that has been termed the worst prediction ever in the history of sciences. Of course, we could imagine that a hitherto unknown symmetry of QG is setting the value of Λ to zero, but then it is hard to imagine why it would not be an exact zero. Why would there be this residual amount? This gives rise to a related philosophical issue. If dark energy is a classical Λ , and hence not changing, why is it so fine-tuned as to be making this transition to becoming dominant now, making ours a very special epoch? This is known as the "Why Now?" puzzle. It turns out that causal set theory not only predicts the correct magnitude for Λ but gives rise to a tracking model that solves the "Why Now?" puzzle. This prediction was made by Rafael Sorkin well before the actual observation of the magnitude of dark energy [62, 63] .

我们似乎可以将暗能量视为宇宙学常数，但细究之下就会发现若干问题。我们预期宇宙学常数在自然单位制中量级为 1，即每个普朗克 4 维体积应当对应普朗克能量。而观测得到的/最佳拟合的能量密度值，比这个理论预期小了约 120 个数量级，这种差异被称为科学史上最糟糕的理论预言。当然，我们可以猜想量子引力中某种迄今未知的对称性将 Λ 的值设为零，但我们很难理解它为什么不是恰好为零。为什么会剩下这一点残余量？这衍生出一个相关的哲学问题: 如果暗能量是经典的 Λ ，因此不随时间变化，那它为什么被精细调节到恰好在现在成为主导成分，让我们所处的时代成为一个非常特殊的纪元？这就是著名的“为何是现在?”难题。而因果集理论不仅能正确预言 Λ 的量级，还给出了一个追踪模型解决了“为何是现在?”难题。这个预言由拉斐尔·索金在实际观测到暗能量的量级 [62, 63] 很久之前就已经提出。

In the next section, we will talk about causal set theory that happens to be a promising solution to the problem of QG. The section after that contains a discussion of how we can naturally accommodate the initial conditions of the early universe in some models that naturally arise in the dynamics of causal sets. The second last section will contain a description of a solution to the problem of the dark energy. We will then bring this chapter to an end with some concluding remarks.

在下一节中，我们将讨论因果集理论，它恰好是量子引力问题一个很有前景的解决方案。在下下节，我们会讨论在因果集动力学中自然涌现的部分模型里，如何自然容纳早期宇宙的初始条件。倒数第二节将介绍暗能量问题的一个解决方案。最后我们会以一些总结性评论结束本章。

Causal Sets

因果集合

A causal set, or "causet" for short, is a locally finite partially ordered set A partially ordered set C consists of a "ground set," which one generally labels with integers from 0 to $N-1$ (N can be infinite), along with a binary relation ($\prec$) which is irreflexive ($x \nprec x$) and transitive ($x \prec y \prec z \Rightarrow x \prec z$) . A causal set, or "causet" for short, is a locally finite partially ordered set whose elements can be thought of as irreducible "atoms of spacetime" (also called "satoms"). Here local finiteness is the condition that every order interval (or simply interval) $[x, y] = \{y \mid x \prec y \prec z\}\ \forall x, z \in C$ has finite cardinality. Just like water in a glass, despite having a finite number of atoms, can be approximated as a continuous fluid, so can be a (part of a) causal set as a (part of a) continuous spacetime. The connection to macroscopic spacetime arises via the notion of a "sprinkling," in which one selects events of a spacetime at random by a Poisson process, identifies them with causal set elements, and then deduces a partial ordering among the elements from the causal structure of the spacetime. One regards a (continuum) spacetime as being a good approximation to an underlying (discrete) causal set if that causal set is likely to have arisen from a sprinkling into that spacetime. For an extensive review of the causal set program, see [29, 64-70].

因果集合简称“因果集”，是局部有限的偏序集。偏序集 C 由一个“基集”(通常用 0 到 $N-1$ 的整数标记，N 可以是无限的)，以及一个满足非自反性 ($x \nprec x$) 和传递性 ($x \prec y \prec z \Rightarrow x \prec z$) 的二元关系 ($\prec$) 构成。因果集合简称“因果集”，是局部有限偏序集，其元素可被视作不可约的“时空原子”(也简称“时原子”)。此处局部有限性指任意序区间 (或简称区间) $[x, y] = \{y \mid x \prec y \prec z\}\ \forall x, z \in C$ 的基数都是有限的。就像杯中的水虽然由有限个原子构成，却可以被近似为连续流体一样，(一部分) 因果集也可以被近似为 (一部分) 连续时空。因果集和宏观时空的关联通过“撒播”的概念建立: 撒播就是通过泊松过程在时空中随机选取事件，将这些事件对应为因果集的元素，再从时空的因果结构推导出元素之间的偏序关系。如果一个因果集很可能是通过撒播进入某 (连续) 时空得到的，那么就可以认为该连续时空是对底层离散因果集的良好近似。关于因果集研究纲领的详尽综述，参见文献 [29, 64-70]。

There are several ways to represent a causet. A graphical representation is called a Hasse diagram, in which elements are represented by vertices and edges depict the relations between these elements. Only irreducible relations are drawn. Figure 1 shows the Hasse diagrams of a ten-element causal set. Elements c and e in this particular causet are not related, whereas elements a, b, and c and d are all related to each other and hence form a chain.

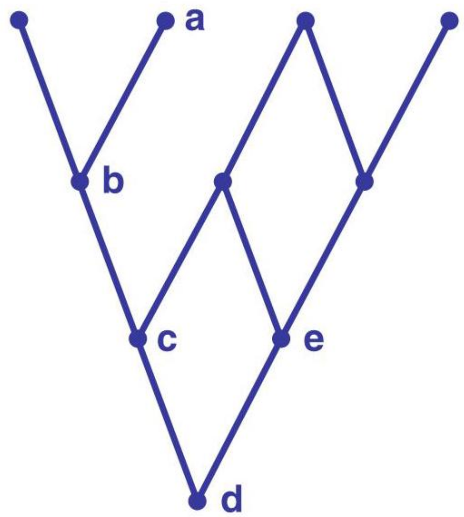
因果集有多种表示方法，其中一种图形表示叫哈塞图: 哈塞图用顶点表示因果集元素，用边表示元素之间的关系，且只绘制不可约关系。图 1 展示了一个含 10 个元素的因果集的哈塞图。在这个特定因果集中，元素 c 和 e 不相关，而元素 a、b、c、d 两两都相关，因此它们构成一个链。

In what follows, some results from the kinematics and the dynamics of causal sets are summarized that we will make use of in the coming sections of this chapter. For details see the references mentioned alongside the summarized results and other chapters of the handbook.

下文将总结我们在本章后续部分会用到的因果运动学与动力学相关结果，详细内容参见总结结果旁给出的参考文献以及本手册的其他章节。

Fig. 1 Hasse diagram of a ten-element causal set. Vertices represent the elements of causal sets and edges represent the relations. If $x \prec y$, then y is drawn above x

图 1 含 10 个元素的因果集的哈塞图。顶点代表因果集元素，边代表关系。若 $x \prec y$ ，则 y 绘制在 x 上方



- In order for a causal set to be likely to arise from a sprinkling, it must be the case that the number of elements sprinkled into any region of spacetime with volume V is Poisson distributed, with a mean of N , where $N = V/l_f^4$. Here l_f is some fundamental length, say, the Planck length. This means that if V is fixed, the expected number of satoms it contains is likely to fluctuate between $N+\sqrt{N}$ and $N-\sqrt{N}$. This Poisson fluctuation in the correspondence between a spacetime volume and the number of elements or satoms it contains plays a crucial role in the prediction of a fluctuating cosmological constant to be discussed later.

> - 若要让一个因果集有概率通过撒播得到，那么撒入任意体积为 V 的时空区域的元素数量必须服从泊松分布，均值为 N ，其中 $N = V/l_f^4$ 。此处 l_f 是基本长度，例如普朗克长度。这意味着如果固定 V ，它包含的时原子预期数量大概率会在 $N+\sqrt{N}$ 到 $N-\sqrt{N}$ 之间涨落。时空体积与其所含元素 (即时原子) 数量对应关系中的这种泊松涨落，对于我们后续讨论的涨落宇宙学常数的预言起到了关键作用。

- A subset of a causal set in which every element is related to every other element is called a chain. The length of a chain is defined as the number of elements it contains minus one. A pair of distinct but related elements in a causal set may have many chains of different lengths between them. In Minkowski space of any dimension, it has been proven that the length of the longest chain between any pair of elements is proportional to the proper time between the events at which they are sprinkled, in the limit of infinite sprinkling density [71,72]. In [73] the proportionality is claimed to hold for any spacetime. Following [72], we define m to be the constant of proportionality, so that

> - 若因果集的一个子集内所有元素两两都相关，则该子集称为链。链的长度定义为它包含的元素数量减一。因果集中一对不同但相关的元素之间可能存在多条长度不同的链。已证明，在无限撒播密度极限下，任意维度闵氏空间中任意一对元素之间最长链的长度与它们被撒入的事件之间的原时成正比 [71,72]。文献 [73] 指出该比例关系对任意时空都成立。参照文献 [72]，我们定义 m 为比例常数，因此有

$$\tau = mL. \tag{1}$$

- The classical sequential growth [74], or CSG for short, is arguably the most developed dynamics for causal sets to date. It describes the causal set as growing via a sort of ”cosmological accretion” process, in which elements of the causet arise one at a time, each selecting some subset of the causal set to be its past. So, for example, we start with an empty universe. Then the first element appears because the universe exists. Then the next element is born. There can be two options at this stage: either the newborn is linked to the already existing element or not. It should be mentioned here that in CSG a new element can never appear to ”the past” of an already existing element. If we wanted, we could label each element in the order it appears. Next, a third element is born either related to both of the preexisting elements, or to either one of them, or to neither (see Fig. 2), and so on. The process of growth in the model is stochastic in the sense that each newborn element selects a ”precursor set” from already existing elements at random, with probabilities which satisfy a discrete analog of general covariance and a causality condition akin to that used to derive the Bell inequalities. This randomness is regarded as fundamental, and yet purely classical in nature, because it does not allow for any quantum interference among alternative outcomes. Given the classical nature of the probability distribution, the dynamics is incomplete, but can be seen as a stepping stone toward formulating a fully quantum process, which could then be regarded as a generalization of classical probability theory. Although the dynamically generated causal sets do not lead to orders which are readily approximated by smooth spacetime manifolds, they do have a number of striking cosmological features, some of which are to be discussed in the next section.

- 经典顺序增长 (classical sequential growth，简称 CSG) 可以说是目前因果集最成熟的动力学。它将因果集描述为通过一种“宇宙吸积”过程增长: 因果集的元素逐次产生，每个元素都会选择因果集的某个子集作为它的过去。例如，我们从一个空宇宙开始，之后第一个元素因宇宙存在而出现，接着下一个元素诞生，这个阶段有两种可能: 新生元素要么与已存在的元素相连，要么不相连。这里需要说明，在 CSG 中，新元素永远不可能出现在已有元素的“过去”。如果需要，我们可以按元素出现的顺序为其标注。接下来第三个元素诞生，它可以和两个已有元素都相关，也可以只和其中一个相关，还可以和二者都不相关 (见图 2)，以此类推。该模型的增长过程是随机的: 每个新生元素从已有元素中随机选择一个“前驱集”，概率满足广义协方差的离散类比，以及与推导贝尔不等式所用条件类似的因果条件。这种随机性被认为是基础性的，但本质上完全是经典的，因为它不允许不同结果之间存在任何量子干涉。鉴于该概率分布的经典性质，这个动力学是不完备的，但它可以作为构建完整量子过程的垫脚石，而完整量子过程可被看作经典概率论的推广。尽管动力学生成的因果集得到的序无法轻易用光滑时空流形近似，但它们确实拥有许多引人注目的宇宙学特征，其中一些将在下一节讨论。

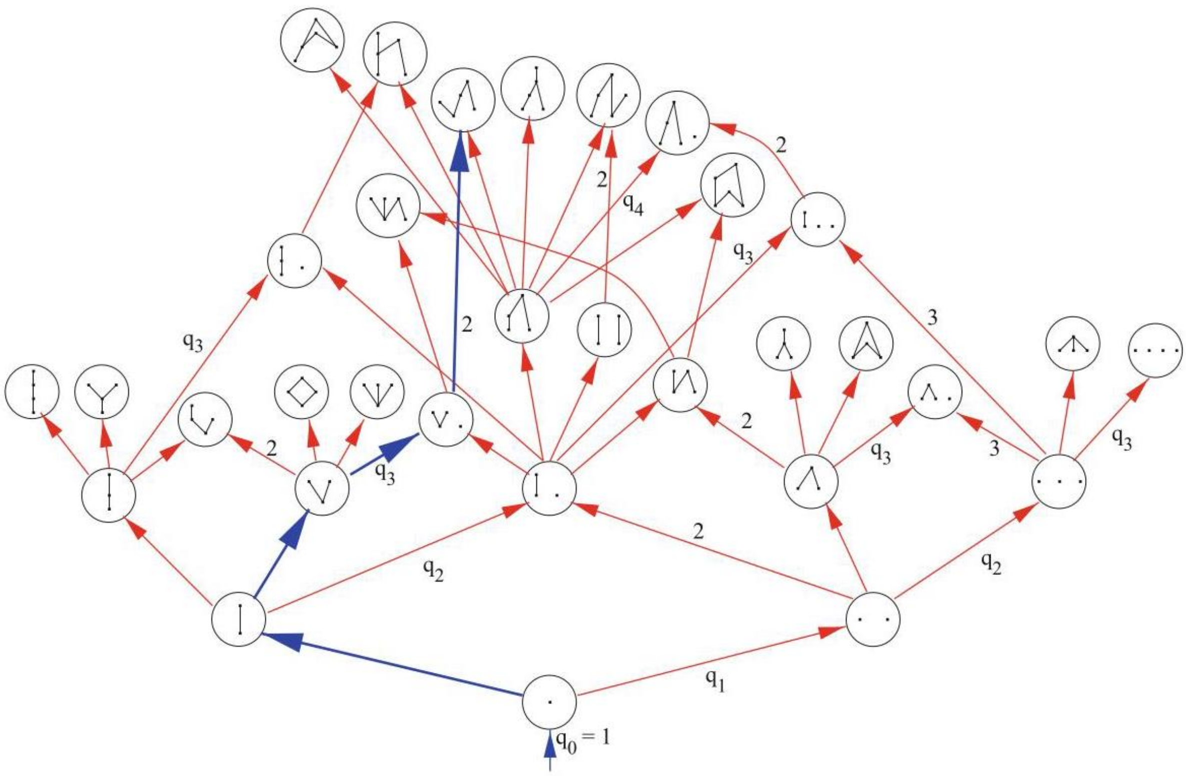


Fig. 2 Starting from, say, a one element causal set, the universe Grows in stages. At each stage, there are several different directions the universe can take and CSG assigns probabilities to all these different possibilities depending on the coupling constant q_i . (Courtesy of D. P. Rideout.)

图 2 举例来说，从一个单元素因果集开始，宇宙分阶段增长。每个阶段宇宙都有多个不同的发展方向，CSG 会根据耦合常数 q_i 为所有这些不同可能性分配概率。(由 D. P. Rideout 提供)

The sequential growth dynamics is described to take place in "stages," where, as mentioned before, it assigns probabilities to all transitions from a given N -element causal set to all "possible" $N+1$ -element causal sets. These probabilities derive from a sequence of nonnegative "coupling constants" $(t_n), n \geq 0$. With these weights, the probability for selecting a precursor set S is proportional to $t_{|S|}$ and is given by

顺序增长动力学被描述为分“阶段”发生，如前所述，它会为从给定 N 元素因果集到所有“可能” $N+1$ 元素因果集的所有跃迁分配概率。这些概率由非负“耦合常数”序列 $(t_n), n \geq 0$ 导出。利用这些权重，选择前驱集 S 的概率与 $t_{|S|}$ 成正比，表达式为

$$\Pr(S) = \frac{t_{|S|}}{\sum_{i=0}^{n} \binom{n}{i} t_i}. \tag{2}$$

Once a precursor is chosen to be the past of the new element, all the relations implied by transitivity are included as well. Thus, it is the "past closure" of S which forms the past of the newly generated element.

当前驱集被选定为新元素的过去后，传递性隐含的所有关系也会被包含进来。因此，正是 S 的“过去闭包”构成了新生成元素的过去。

- The particular sequence $t_n = t^n$, for a single nonnegative real number t , gives rise to a dynamics called transitive percolation [74]. This sequence plays an important role, as we will see in a moment. The rule for deciding which elements to select for the past of a new element is particularly simple for transitive percolation. The newborn element simply considers each already existing element in turn and selects it to be to its past with a fixed probability $p = t/(1+t)$. It then adds to its past every element which precedes any of the originally selected elements, to maintain transitivity.

- 对于单个非负实数 t ，特定序列 $t_n = t^n$ 会生成一种称为传递渗流的动力学 [74]。我们很快就会看到，该序列发挥着重要作用。对于传递渗流，判断为新元素的过去选择哪些元素的规则格外简单: 新生元素只需依次考察每个已有元素，以固定概率 $p = t/(1+t)$ 将其选入自己的过去，之后再把所有排在最初所选元素之前的元素都加入自己的过去，以保持传递性。

- An element of a causal set which is related to every other element of the causal set is called a ”post.” This would resemble an initial or final singularity of a universe, in that the entirety of the universe is causally related to it.

- 如果因果集中的一个元素与因果集内所有其他元素都相关，就称它为一个“极点 (post)”。这类似于宇宙的初始奇点或最终奇点，因为整个宇宙都与它存在因果关联。

- It has been shown that a ”large” class of CSG models lead to causets which almost surely contain an infinite number of posts [75]. For example, the sequence $t_i = (\alpha/\ln i)^i$ for $i > 0, \alpha > \pi^2/3$ shows this behavior. Now the presence of posts in the dynamical model suggests an interesting possibility, as described in [76], namely, that either we can think of the universe after the post as the old universe with the same coupling constants or we can forget the universe before the post and think of the part after the post as an entirely new universe but with coupling constants which are ”renormalized” with respect to those of the previous era. Much is now known about the flow of the coupling constants (t_n) under this ”cosmic renormalization” [77, 78]. The most important result for us is that transitive percolation belongs to this class of dynamical models that generate posts and in particular that it forms a unique attractive fixed point in this space. The sequence $t_n = t^n/n!$ has also been studied in some detail [76, 79] . There it is shown that the region immediately subsequent to a post behaves like transitive percolation, with a parameter t which gets driven toward zero under the cosmic renormalization $t \to \sqrt{t/N}$ for N elements to the past of the current era’ s post (or ”origin element”).

- 研究表明，一大类 CSG 模型生成的因果集几乎必然包含无穷多柱点 [75]，例如序列 $t_i = (\alpha/\ln i)^i$ 在 $i > 0, \alpha > \pi^2/3$ 条件下就呈现出这种行为。正如文献 [76] 所述，动力学模型中柱点的存在引出了一个有意思的可能性: 我们既可以将柱点之后的宇宙视为保留原有耦合常数的旧宇宙，也可以忽略柱点之前的宇宙，将柱点之后的部分看作一个全新宇宙，只不过它的耦合常数相对前一时代发生了” 重整化”。目前人们已经充分了解耦合常数 (t_n) 在这种” 宇宙重整化” 下的流动行为 [77, 78]。对我们而言最重要的结论是: 传递渗落属于这类会生成柱点的动力学模型，且它在该模型空间中形成一个唯一吸引不动点。序列 $t_n = t^n/n$ 也已经得到了较为细致的研究 [76, 79] 。研究表明，柱点之后紧邻区域的行为和传递渗落一致，其参数 t 在宇宙重整化下被驱动趋近于零，$t \to \sqrt{t/N}$ 对应当前时代柱点 (即” 原点元”) 过去方向的 N 个元素。

The Early Universe of Causal Set Theory

因果集理论的早期宇宙

We have already mentioned that transitive percolation generates a cyclic universe. We will argue in this section that the initial part of a ”new cycle,” i.e., the part of the causet generated by percolation right after a post, gives rise to an early universe that can naturally answer the two questions raised in the introduction to this chapter; why is the early universe so large when it is not so old and why is it so homogeneous when it has so many causally disconnected parts? A popular way to solve these puzzles is to fine-tune (the potential of) a hypothetical scalar field so that it causes a very small part of the early universe to undergo an exponential expansion very early on. Reference [80] argues that percolation does that naturally without any fine-tuning. It generates an early universe that not only ”becomes large” (the sense of spatial size is discussed later in this section) very early on but is also very homogeneous. Let us discuss the early universe thus produced in a bit more detail.

我们已经提到，可传递渗流会生成循环宇宙。本节我们将探讨，“新循环”的初始部分，即大爆炸之后刚通过渗流生成的因果集部分，如何产生能自然解答本章引言提出的两个问题的早期宇宙: 为何早期宇宙年龄不大但体量却如此庞大，为何它拥有这么多因果不连通区域却依然如此均匀? 解决这些谜题的一种主流方法是对假想标量场 (的势) 进行精细调节，使早期宇宙极小的一块区域在极早期发生指数膨胀。文献 [80] 指出，渗流无需任何精细调节就能自然实现这一点: 它生成的早期宇宙不仅极早期就“变得庞大”(空间尺度的含义将在本节后半部分讨论)，而且还十分均匀。接下来我们稍微详细地讨论一下由此产生的早期宇宙。

Originary Percolation

起源渗流

Note that although the causal set produced by the transitive percolation is sure to have posts, an arbitrary stage in that causal set is almost guaranteed not to be one. Thus, if we want to study the universe that follows a post, we need an effective dynamics that produces it and the causal set that follows. In other words, we need to find an effective dynamics that produces a causal set as if produced by the transitive percolation but for which the possibility of being born unrelated to already existing elements is excluded. This is provided by the ”originary percolation” where the probabilities are the same as those of an ordinary transitive percolation but conditioned on the event that the newborn element connects to at least one other element. We simply normalize all other probabilities. Note that this can be done with any CSG model. Originary percolation is simply the originary version of transitive percolation. The originary dynamics is in fact one of the general classes of solutions to the covariance and causality conditions on sequential growth, described in [81].

请注意, 虽然传递渗流生成的因果集必然存在 posts, 但该因果集中的任意阶段几乎都不可能是 post。因此，如果我们想要研究 post 之后的宇宙，就需要一种能生成该 post 及其后续因果集的有效动力学。换句话说，我们需要找到一种有效动力学，它生成的因果集满足传递渗流的生成结果，但排除了新生成元素与已有元素无关联的可能性。这一需求由「起源渗流」满足: 起源渗流的概率与普通传递渗流相同，但约束新生成元素至少连接一个其他元素。我们只需对所有其他概率做归一化处理即可。注意任何 CSG 模型都可以这样处理，起源渗流就是传递渗流的起源版本。实际上，起源动力学是序贯增长满足协方差与因果性条件的一类通解，已在文献 [81] 中描述。

How is this effective dynamics different from the one originally described? Much of it is still the same. As mentioned earlier, at each stage of the growth process, the newborn element considers each existing element x in turn and selects x to be in its past with probability p . In order to maintain transitivity of the order, if it chooses x for part of its past, it includes all ancestors of x as well. The change is that in the event that no element x is selected in this process, the newborn simply ”tries again,” so as to maintain the condition of originarity. At stage n (meaning that there are n elements currently in the causet), the probability to select a particular subset S of existing elements is

这种有效动力学和最初的传递渗流动力学有何区别? 大部分性质仍然相同。如前所述，在增长过程的每个阶段，新生成元素依次考察每个已有元素 x ，并以概率 p 将 x 归入自己的过去。为了保持序的传递性，如果选择 x 作为自身过去的一部分，那么也会将 x 的所有祖先都纳入。两者的区别在于: 如果在这个过程中没有选中任何元素 x ，新元素就会直接「重试」，以此满足起源条件。在阶段 n (即当前因果集中共有 n 个元素)，选中某个特定现有元素子集 S 的概率为

$$\Pr(S) = \frac{p^{|S|} q^{n-|S|}}{1 - q^n}, \tag{3}$$

where $q = 1 - p$ and the factor $1/(1 - q^n)$. This conditional probability accounts for the originary condition, which excludes the possibility of not connecting to anything (which occurs with probability q^n). Once a set S is chosen, the past closure of S becomes the past of the newborn element x .

其中为 $q = 1 - p$ ，因子为 $1/(1 - q^n)$ 。这个条件概率体现了起源条件，排除了不连接任何元素的情况 (该情况发生的概率为 q^n)。选定集合 S 后，S 的过去闭包就成为新元素 x 的过去。

The early universe produced by the originary percolation happens to have two phases. It first starts into a random tree phase and then transitions into a de Sitter-like phase. We now discuss them in turn.

起源渗流生成的早期宇宙恰好存在两个阶段: 它首先进入随机树阶段, 随后转变为类德西特阶段。我们接下来依次讨论这两个阶段。

Random Tree Phase

随机树阶段

In transitive percolation, at stage n, the probability of connecting the new elements with m already existing elements is

在传递渗流中，第 n 阶段，新元素与 m 个已存在元素建立连接的概率为

$$\Pr(|S| = m) = \binom{n}{m} p^{|S|} q^{n-|S|} \tag{4}$$

which is vanishingly small for any $m > 0$ for $p \ll 1$. This means that initially the elements tend to be born in isolation and the very early universe looks like an antichain with high probability, where an antichain is a causal set where no two elements are connected. Since we eliminate this possibility in originary percolation and condition the rest of the probabilities to this event, the abovementioned expression becomes

当 $p \ll 1$ 满足条件时，该概率对任意 $m > 0$ 都极小。这意味着初始时元素往往孤立诞生，极早期宇宙极高概率呈现为反链——反链即任意两个元素都无连接的因果集。由于我们在原生渗流中排除了这种可能性，并将其余概率以该事件为条件进行修正，上述表达式变为

$$\Pr(|S| = m) = \binom{n}{m} \frac{p^{|S|} q^{n-|S|}}{1 - q^n}, \tag{5}$$

and we get the result that most of the elements in the beginning have, instead, just one parent element. These elements are said to form a random tree (Fig. 3). It persists until $np \sim 1$ or $n \sim 1/p$, at which point we enter a new phase, which heralds the end of the random tree era, and the beginning of a de Sitter-like expansion, which we will discuss later in this subsection. Note that the fact that initially we will have a tree does not depend on p as long as $n \ll 1/p$. In other words, the structure of this earliest random tree era of the universe is practically independent of p , save in determining how long it lasts.It should be intuitively clear that the ”spatial” size of the universe increases ”quickly” during the random tree era. We can take the cardinality of the set of elements at level t to be a measure of the size of the universe at time t , where the aforementioned set consists of all elements that are connected to the initial (or root) element such that the longest chain between them is of length t . This question has been discussed in the combinatorics literature, under the name ”random recursive trees” [82]. In reference [80], the authors observe that, after forming a random tree with N elements, the mean cardinality of level t looks very much like a multipole of a Poisson distribution in t . Thus, the mean number N_t of elements in level t is very well fit by the function

我们得到的结论是，初始阶段大多数元素反而仅拥有一个父元素。这些元素构成了随机树 (图 3)。随机树阶段会持续到 $np \sim 1$ 或 $n \sim 1/p$，此时我们进入新阶段，它标志着随机树时代的结束，类德西特膨胀的开始，我们将在本小节稍后讨论这一膨胀。需要注意，只要满足 $n \ll 1/p$，初始形成树这一结论与 p 无关。换言之，宇宙最早的随机树时代的结构几乎不依赖 p，p 仅决定该阶段持续的时长。根据直觉不难理解，宇宙的“空间”尺度在随机树时代会“快速”增长。我们可以将 t 层的元素集合的基数作为 t 时刻宇宙尺度的度量，这里 t 层的元素集合指所有与初始 (根) 元素相连，且二者之间最长链长度为 t 的元素的集合。这个问题已经在组合数学文献中以“随机递归树”的名称被讨论过 [82]。在文献 [80] 中，作者观察到，在生成包含 N 个元素的随机树后，t 层的平均基数极近似于关于 t 的泊松分布多项式。因此，t 层元素的平均数量 N_t 可以很好地用下式拟合:

$$N_t = \frac{A\lambda^t e^{-\lambda}}{t!}, \tag{6}$$

where the normalization factor $A \sim N$ and $\lambda \sim \ln N$. An example is shown in Fig. 4 which has been taken from [80], as are all other figures in this section. It is worth noting that despite the excellent fit of Fig. 4, the relation cannot be exact, for example, because N_t must be exactly zero for $t > N$, which does not occur in (6).

其中归一化因子为 $A \sim N$ 和 $\lambda \sim \ln N$。实例如图 4 所示，图片源自文献 [80]，本节其余图片也均来自该文献。值得注意的是，尽管图 4 的拟合效果极佳，该关系并不精确，例如，当 $t > N$ 满足条件时 N_t 必须严格为零，这在式 (6) 中无法成立。

Fig. 3 A sample causal set generated by originary percolation with $N = 16$, $p = 0.2$. Elements of the initial tree, which are elements with only one parent, are shown by red squares

图 3 由原生渗流生成的样本因果集，参数为 $N = 16$，$p = 0.2$。初始树的元素 (即仅拥有一个父元素的元素) 用红色方块标出

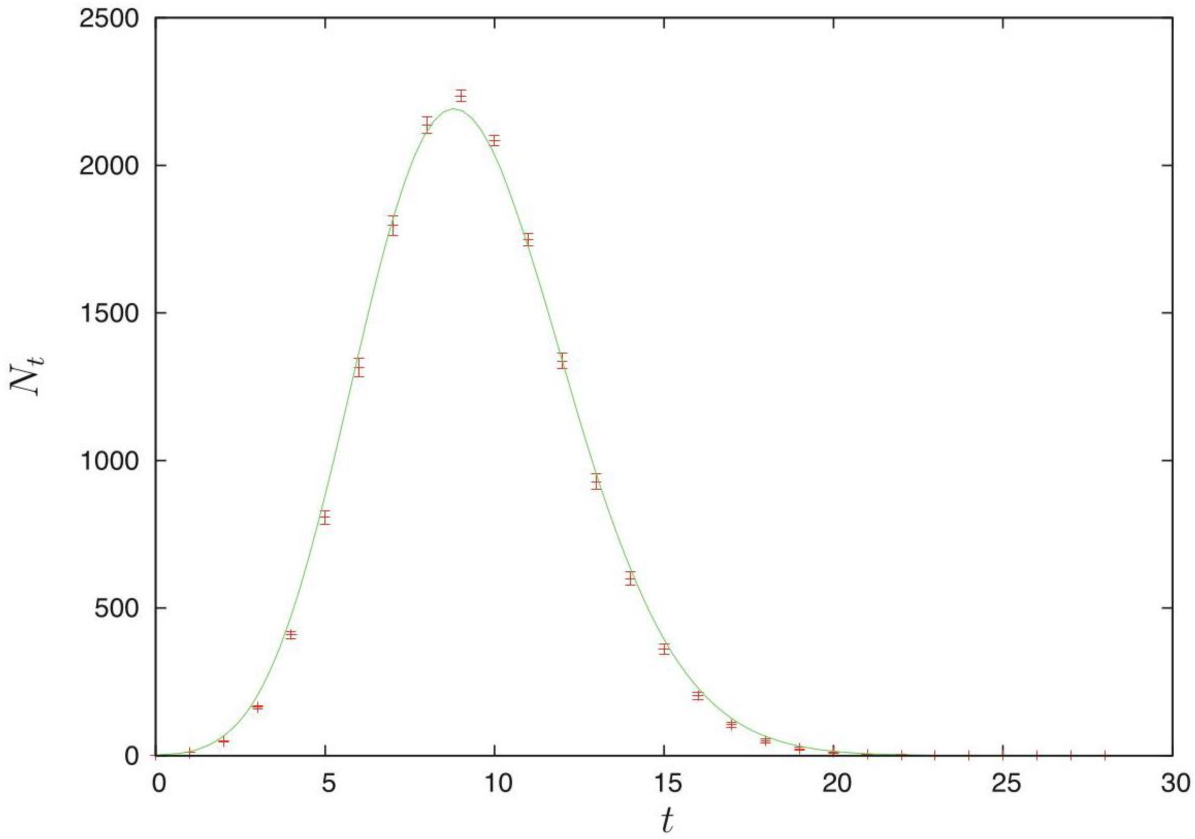


Fig. 4 Mean population of levels (defined in the text) for an $N = 16384$ element random tree. The mean and its error are measured from 100 samples of the random tree process. The fitting function proportional to a Poisson distribution is shown. Here $A = 16687 \pm 85$ and $\lambda = 9.306 \pm .022$

图 4 含 $N = 16384$ 个元素的随机树各层 (定义见正文) 的平均元素数量。均值和误差由对随机树过程的 100 次采样测得，图中给出了与泊松分布成正比的拟合函数。参数为 $A = 16687 \pm 85$ 和 $\lambda = 9.306 \pm .022$

Rate of Expansion

膨胀率

In order to get an idea of the rate of expansion in this early universe generated by originary percolation, we look at the cardinality of levels in the sense defined above.

为了了解原生渗流生成的早期宇宙的膨胀速率，我们按照上文定义考察了各层的基数。

Figure 5 shows the mean cardinality of different levels in ten causets that were generated using originary percolation at $p = .001$. The sets have $N = 11585$. The cardinality of each level is given both for the random tree era (blue points) and for the whole causet (red points). In addition to the cardinality of each level, the cardinality of a ”foliation” of the causal set by inextendible antichains is also depicted in the figure (green points). Here an inextendible antichain is one which is maximal in the sense that no elements can be added to it while remaining an antichain, i.e., every other element of the causal set is to the future or past of one of its elements. This is a more appropriate analog to the (edgeless) spatial hypersurfaces of general relativity. For a detailed discussion of how the antichains used in the figure are defined, see [80]. In addition, the number of elements of the initial tree in each level (blue points) and the number of maximal elements of the initial tree in each level (magenta points) are also shown.

> 图 5 展示了在 $p = .001$ 条件下通过原生渗流生成的 10 个因果集不同层次的平均基数。这些集合具有 $N = 11585$ 。图中同时给出了随机树时期 (蓝点) 和整个因果集 (红点) 各层的基数。除各层基数外，图中还绘制了因果集通过不可延反链生成的“叶状结构”的基数 (绿点)。此处不可延反链是指无法再添加元素仍保持反链性质的极大反链，即因果集中所有其他元素都位于其某一元素的未来或过去方向。它是广义相对论中 (无边界) 空间类超曲面更贴切的类比。关于图中所用反链的定义的详细讨论，参见文献 [80]。此外，图中还绘制了各层初始树的元素数量 (蓝点) 以及各层初始树的极大元素数量 (品红点)。

It can be seen from the above discussion that the spatial size of the early universe after a post generated by percolation grows very quickly. The smaller the p , the larger is the early universe. As there are models that behave like percolation early on but where effective p naturally becomes smaller and smaller with each cycle, there is no fine-tuning involved, at least, in this sense.

> 从上述讨论可以看出，渗流生成一个新节点后，早期宇宙的空间尺寸增长非常迅速。p 越小，早期宇宙的规模就越大。由于部分模型在早期的行为和渗流类似，且有效 p 会随每个周期自然逐渐减小，因此至少在这个层面上不需要精细调谐。

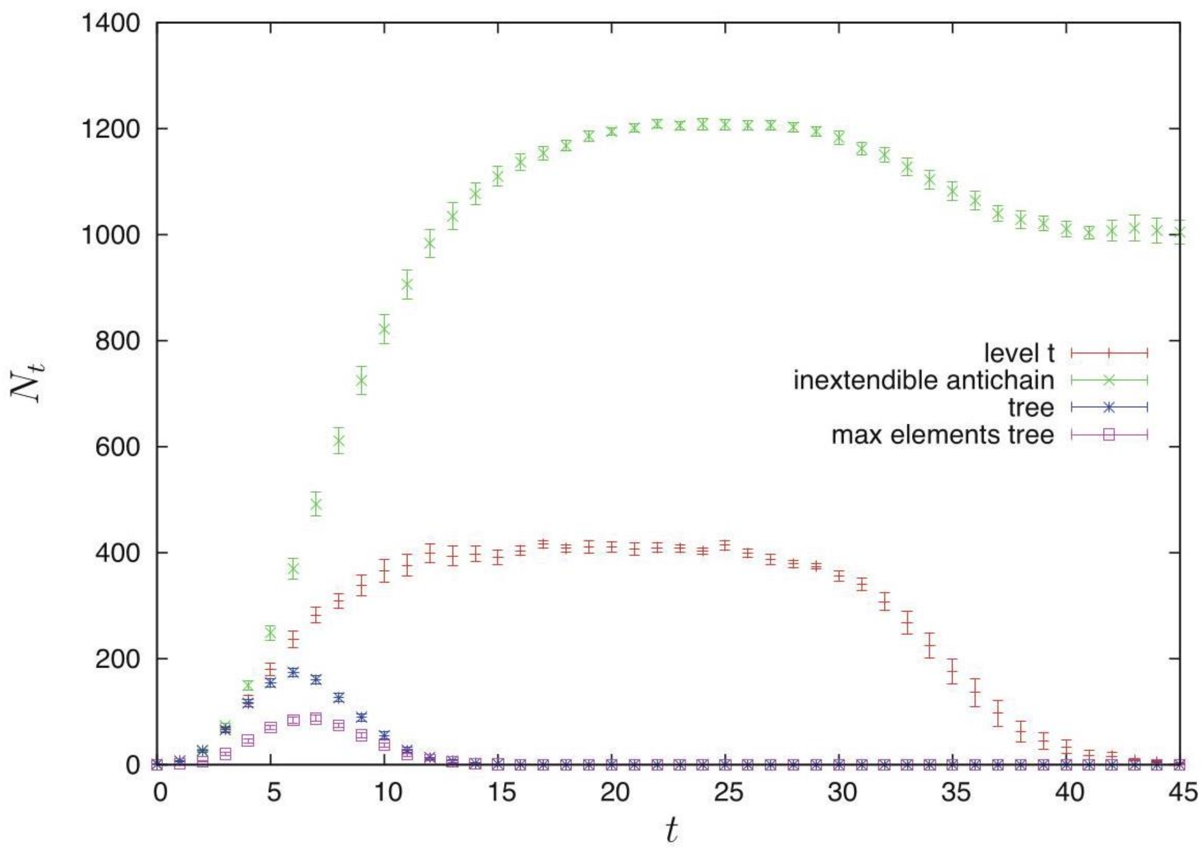


Fig. 5 Mean ”spatial volume” of 10 originary percolated causal sets with $N = 11585, p = .001$. Shown is the cardinality of level t in red, the cardinality of an inextendible antichain containing level t in green, the number of elements of the initial tree at level t in blue, and the number of maximal elements of the initial tree, at level t , in magenta

图 5 10 个原生渗流因果集在 $N = 11585, p = .001$ 下的平均“空间体积”。红色为层 t 的基数，绿色为包含层 t 的不可延反链的基数，蓝色为层 t 处初始树的元素数量，品红色为层 t 处初始树极大元素的数量

The other extremely important fact to notice is that the this early universe is homogeneous in the sense that the future of every element of a percolated causal set is itself an instance of originary percolation. In other words, the future of an element is the same (in probability) as that of any other element. Note that since we are discussing the future of an element x , we have a set of elements that are all joined to x . This is exactly the condition of originary percolation. Thus, originary percolation describes a homogeneous universe, for which the future of every element is exponentially expanding. Now a homogeneous and exponentially expanding spacetime reminds us of de Sitter spacetime. Does this early universe look anything like de Sitter spacetime? This question takes us into the discussion of the next phase after the random tree era. We will present evidence that suggests this link quite strongly.

另外需要注意的一个极其重要的事实是: 这个早期宇宙是均匀的，因为渗流因果集中任意元素的未来本身就是一个原生渗流实例。换言之，任意元素的未来在概率上和其他任何元素的未来都是相同的。注意，我们讨论元素 x 的未来时，所有得到的元素都与 x 相连，这正好满足原生渗流的条件。因此，原生渗流描述的是一个均匀宇宙，其中每个元素的未来都呈指数膨胀。均匀且指数膨胀的时空让我们联想到德西特时空。这个早期宇宙是否和德西特时空有相似之处？这个问题将我们引向随机树时期之后下一阶段的讨论，我们会给出证据，有力地支持二者的关联。

The de Sitter Phase

德西特相

The initial universe, or at least the one that can be simulated on a computer, does not have enough elements or satoms for the familiar notions of the continuum to be rendered properly. This is particularly clear if one considers, for example, the random tree era. It is not possible to define the notion of ”spacelike distance” in a random tree as no two elements have a common future [83]. On the other hand, the notions of the length of the longest chain, L , between two related causal set elements, which, as mentioned before we take to be proportional to the proper time between the two events, τ , is still properly defined. Similar is the case for the number of causal set elements, $N_\diamond$, which are causally between two given elements and which is proportional to the volume of the Alexandrov neighborhood, $V_\diamond$, formed by the two elements. More correctly we define $N_\diamond = |\{z \mid x \prec z \prec y\}| + 1$ for an order interval $[x, y]$ where $|\cdot|$ indicates set cardinality. The $+1$ allows $N_\diamond = L$ for an interval which is a chain. The Alexandrov neighborhood of two events, on the other hand, is the overlap of the past of the future-most event with the future of the other. As de Sitter space is ”spherically symmetric,” we can represent the Alexandrov neighborhood of two events in t and r space as sketched in Fig. 6. We then try to see if L and $N_\diamond$ for pairs of satoms in originary percolation follow the same relationship as τ and $V_\diamond$ in $D + 1$ -dimensional de Sitter spacetime. Since this τ and $V_\diamond$ relationship should be very specific to not only the space being de Sitter but of a particular D as well, a good fit between the two should be an excellent indicator of whether or not de Sitter is a good approximation for the part of percolation right after the post.

> 初始宇宙，或者至少是能在计算机上模拟的初始宇宙，没有足够的元素 (即源原子) 来恰当呈现我们熟悉的连续统概念。例如，在随机树时期这一点格外明显。我们无法在随机树中定义“类空间距”的概念，因为随机树中没有两个元素拥有共同未来 [83]。另一方面，最长链长度，即 L ，是两个存在因果关联的因果集元素之间的量，如前文所述，我们认为它正比于两个事件之间的固有时，即 τ ，这个概念仍然是良定义的。类似地，位于两个给定元素之间的因果元素的数量，即 $N_\diamond$ ，它正比于由这两个元素构成的亚历山德罗夫邻域的体积，即 $V_\diamond$ ，这个概念也是良定义的。更准确地说，我们为序区间 $[x, y]$ 定义 $N_\diamond = |\{z \mid x \prec z \prec y\}| + 1$ ，其中 $|\cdot|$ 表示集合的基数。+1 项使得该定义适用于本身就是链的区间 $N_\diamond = L$ 。另一方面，两个事件的亚历山德罗夫邻域，是最远未来事件的过去与另一个事件的未来的交集。由于德西特空间是“球对称”的，我们可以将两个事件的亚历山德罗夫邻域绘制在 t 和 r 空间中，如图 6 所示。我们接下来尝试检验，原生渗因果集中源原子对的 L 和 $N_\diamond$ 是否遵循与 $D+1$ 维德西特时空中 τ 和 $V_\diamond$ 相同的关系。由于该 τ 和 $V_\diamond$ 关系不仅对德西特空间是特定的，还取决于具体的 D ，因此两者若拟合良好，就能充分说明德西特空间是渗变后阶段的良好近似。

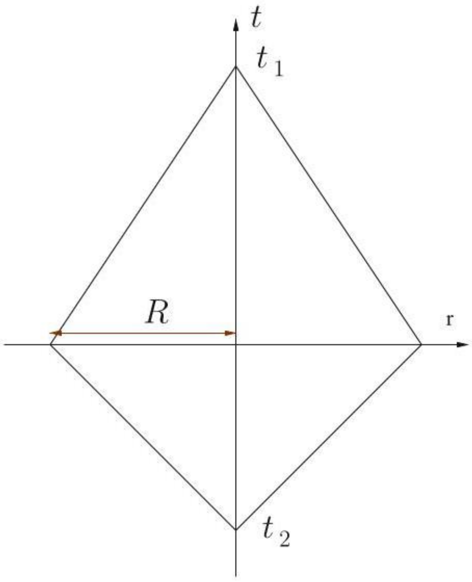
The L and $N_\diamond$ relationship for a causet generated by originary percolation can be easily calculated on a computer, whereas τ and $V_\diamond$ relationship for a $D+1$ -dimensional de Sitter spacetime with the metric

> 原生渗流生成的因果集的 L - $N_\diamond$ 关系可以很容易在计算机上计算，而带度量的 $D+1$ 维德西特时空的 τ - $V_\diamond$ 关系则由下式给出

$$ds^2 = -dt^2 + e^{2t/\ell}\left(dr^2 + r^2 d\Omega_D^2\right) \tag{7}$$

Fig. 6 An Alexandrov neighborhood (order interval) in de Sitter space. Here $t_1 + t_2 = \tau$ and $R = \ell\left(\frac{e^{\tau/\ell}-1}{e^{\tau/\ell}+1}\right)$.$\ell$ is the so-called radius of curvature of the de Sitter space

> 图 6 德西特空间中的亚历山德罗夫邻域 (序区间)。此处 $t_1 + t_2 = \tau$ 和 $R = \ell\left(\frac{e^{\tau/\ell}-1}{e^{\tau/\ell}+1}\right)$.$\ell$ 是德西特空间的所谓曲率半径



is given by

为

$$V_\diamond = \mathcal{C}_D \ell^{D+1}\left[\ln\cosh^2\left(\frac{\tau}{2\ell}\right) + \sum_{i=1}^{D}\frac{(-1)^{i+1}}{i}\binom{D}{i}\left(\left(1+\tanh\left(\frac{\tau}{2\ell}\right)\right)^i \right.\right.$$
$$\left.\left. + \left(1-\tanh\left(\frac{\tau}{2\ell}\right)\right)^i - 2\right)\right] \tag{8}$$

for D odd and

当 D 为奇数时,

$$V_\diamond = \mathcal{C}_D \ell^{D+1}\left[\frac{\tau}{\ell} + \sum_{i=1}^{D}\frac{(-1)^{i}}{i}\binom{D}{i}\left(\left(1+\tanh\left(\frac{\tau}{2\ell}\right)\right)^i - \left(1-\tanh\left(\frac{\tau}{2\ell}\right)\right)^i\right)\right]$$

(9)

for D even [80]. Here ℓ is the radius of curvature, $\mathcal{C}_D$ is the volume of a D-dimensional unit ball, and all other symbols have their usual meaning [11].

当 D 为偶数时 [80]。此处 ℓ 是曲率半径，$\mathcal{C}_D$ 是 D 维单位球的体积，其余符号均为常规含义 [11]。

One obvious case of interest is $D = 3$. Using the above mentioned expressions and the fact that $\mathcal{C}_3 = 4\pi/3$, it turns out that

一个值得关注的明显情况是 $D = 3$ 。利用上述表达式以及 $\mathcal{C}_3 = 4\pi/3$ ，可以得到

$$V_\diamond = \frac{4\pi}{3}\ell^4\left(\ln\cosh^2\left(\frac{\tau}{2\ell}\right) - \tanh^2\left(\frac{\tau}{2\ell}\right)\right). \tag{10}$$

for a four-dimensional de Sitter spacetime. It should be noted that $V_\diamond \sim \tau^{D+1}$ for $\frac{\tau}{\ell} \ll 1$ as every spacetime looks locally like the Minkowski space (of the same dimension) and $\sim \tau$ for $\frac{\tau}{\ell} \gg 1$. For the four-dimensional de Sitter Space $V_\diamond = \frac{\pi}{24}\tau^4 + O\left(\tau^5\right)$ for $\tau \ll \ell$ and $\approx 4\pi/3\,(\tau - \ln 4e)$ for $\tau \gg \ell$.

对于四维德西特时空成立。需要注意的是，对于 $\frac{\tau}{\ell} \ll 1$ 有 $V_\diamond \sim \tau^{D+1}$ ，因为任何时空局部都等同于同维度的闵氏空间，且对于 $\frac{\tau}{\ell} \gg 1$ 有 $\sim \tau$ 。对于四维德西特空间，当 $\tau \ll \ell$ 时 $V_\diamond = \frac{\pi}{24}\tau^4 + O\left(\tau^5\right)$ ，当 $\tau \gg \ell$ 时 $\approx 4\pi/3\,(\tau - \ln 4e)$ 。

As far as the investigation of L vs $N_\diamond$ is concerned, it is only constrained by the computational power available. The primary constraint is the determination of the longest chain in an interval which involves an $O\left(N^2\right)$ algorithm, and there are $O\left(N^2\right)$ intervals to check. Figure 7 shows the L vs $N_\diamond$ relationship for three causal sets generated by originary percolation with $N = 500, 1500$, and 2500 and at $p = 0.025$. The figure shows the typical behavior of causets generated with different N but with the same p . Before the results are discussed, the following points should be noticed.

就 L 与 $N_\diamond$ 的研究而言，它仅受现有计算能力的限制。主要限制在于确定区间内的最长链需要用到 $O\left(N^2\right)$ 算法，并且共有 $O\left(N^2\right)$ 个区间需要检验。图 7 展示了三个由本原渗流生成的因果集的 L - $N_\diamond$ 关系，生成参数为 $N = 500, 1500$ 、2500，且在 $p =$ 处取值为 0.025。该图展示了不同 N 生成但拥有相同 p 的因果集的典型行为。在讨论结果前，需要注意以下几点。

- It should be obvious, especially from the last figure, that L values for the same $N_\diamond$ value form a small range and vice versa. We will denote the largest $N_\diamond$ value for a given L by $N_\diamond^{\max}(L)$. These form a unique set for a given causet but can change slightly for another causet with the same p and N .

- 尤其从上一图中可以明显看出，相同 $N_\diamond$ 值对应的 L 值分布在一个很小的区间内，反之亦然。我们将给定 L 对应的最大 $N_\diamond$ 值记为 $N_\diamond^{\max}(L)$ 。对于给定因果集，这些值构成唯一集合，但对于拥有相同 p 和 N 的另一因果集，该集合会发生微小变化。

- As N increases, one might expect $N_\diamond^{\max}(L)$ to keep increasing without limit, but it can be seen from the last figure that after a point, the maximum $N_\diamond(L)$ no longer increases with N . It should be obvious that this is necessary if the relationship between L and $N_\diamond$, as with anything else of interest, is to depend solely on p and to be independent of N . It is also true that originary percolation for fixed p and infinite N contains every finite partial order as a suborder. Thus, somewhere in that infinite causet is an interval with height L and cardinality arbitrarily large. However, we do not send $N \to \infty$ for fixed p ; we are only interested in the "early universe" of originary percolation, with N no larger than say $1/p^3$. It can be seen that in such a regime, the maximum $N_\diamond$ is effectively independent of N .

- 随着 N 增大，人们可能会认为 $N_\diamond^{\max}(L)$ 会无限制持续增长，但从上一图可以看出，超过某一节点后，最大 $N_\diamond(L)$ 不再随 N 增长。显然，如果 L 与 $N_\diamond$ 的关系和其他我们关心的性质一样，仅依赖于 p 且与 N 无关，那么这种情况就是必然的。对于固定 p 和无穷大 N 的本原渗流，确实任何有限偏序都可作为其子序存在。因此，在这个无穷因果集中，必然存在一个区间，其高度为 L 且基数任意大。但是，我们并不固定 p 来研究 $N \to \infty$ ，我们仅关注本原渗流的“早期宇宙”阶段，此时 N 不超过 $1/p^3$ 。可以看到，在该区域内，最大 $N_\diamond$ 实际上与 N 无关。

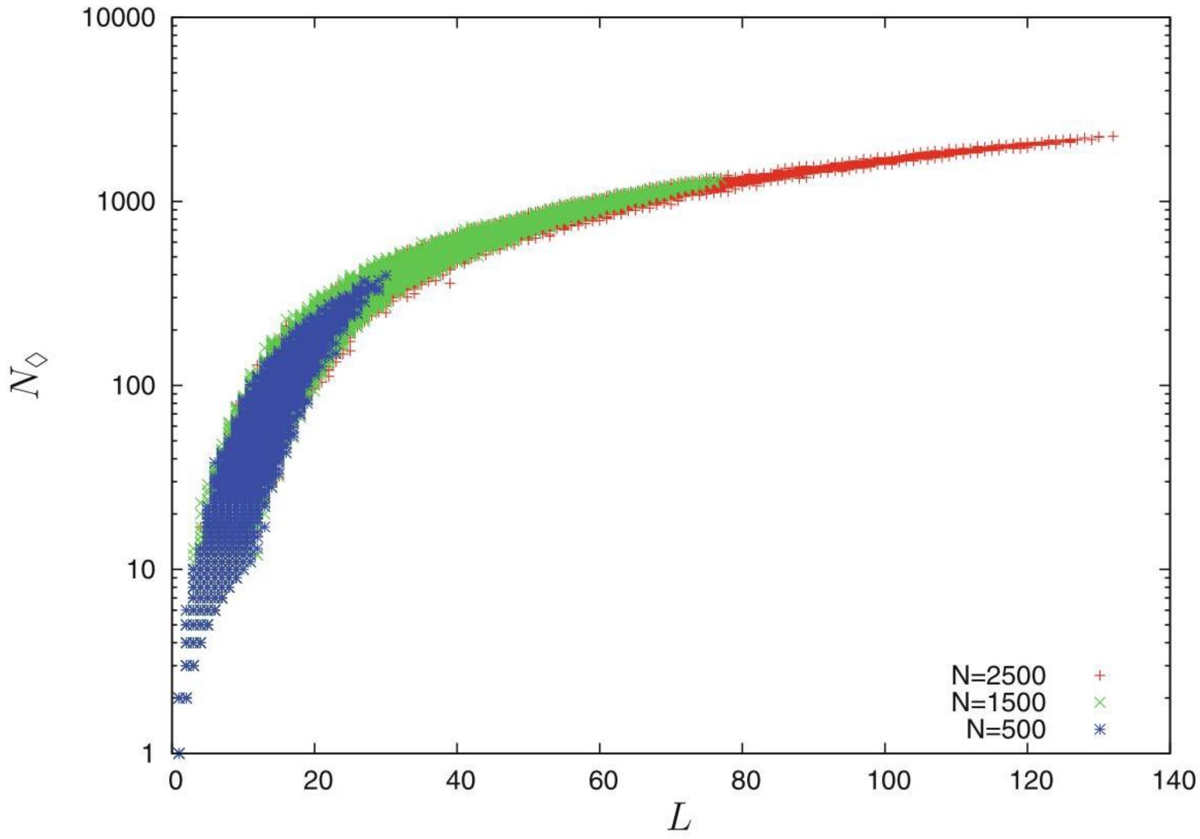


Fig. 7 The set of all pairs $(L, N_\diamond)$ for each related pair of elements in three causal sets generated by originary percolation. Each causet was produced with the same value of $p = .025$ but three different values of $N = 500, 1500$, and 2500. (To make the figure size manageable, we plot only every 4^{th} point for $N = 500$, every 10^{th} for $N = 1500$, and every 200^{th} point for $N = 2500$.)

图 7 三个由本原渗流生成的因果集中，每个相关元素对对应的所有点对 $(L, N_\diamond)$ 的集合。每个因果集均在相同的 $p = .025$ 值下生成，但 $N = 500, 1500$ 取三个不同值，大小为 2500。(为控制图幅，我们仅对 $N = 500$ 每 4^{th} 个点绘制一个点，对 $N = 1500$ 每 10^{th} 个点绘制一个点，对 $N = 2500$ 每 200^{th} 个点绘制一个点。)

- We do not expect the L vs $N_\diamond$ relationship to behave like de Sitter for the smaller values of L as they get a significant contribution from the random tree era. Blue points in Fig. 7 for smaller values of L show this contribution.

- 我们不认为较小 L 对应的 L - $N_\diamond$ 关系符合德西特行为，因为它们会受到随机树阶段的显著影响。图 7 中较小 L 对应的蓝色点就体现了这种影响。

- For a dataset, with a given N, the $N_\diamond^{\max}(L)$ values are smaller than they ought to be for the largest few L values. It can be seen in Fig. 7 where blue points do not match the green ones for the last few L values, and the green points in turn, behave the same relative to the red ones near their end values. This is a result of the finite size of the causal set.

- 对于给定 N 的数据集，最大的少数几个 L 值对应的 $N_\diamond^{\max}(L)$ 值会小于理论值。这一点可以在图 7 中看到: 对于最后几个 L 值，蓝色点与绿色点不吻合，而绿色点在靠近其端点区域，也相对于红色点表现出相同的行为。这是因果集尺寸有限带来的结果。

- As alluded to earlier, almost all of the physics in this scenario is dictated by the choice of p , as long as $N \gg 1/p$. This can be easily seen from Fig. 7, by observing, that (a) the data points for smaller N are effectively a subset of those for a larger value of N and (b) the maximum (and minimum) values of $N_\diamond$ for the $N = 1500$ causet are the same as those for the $N = 2500$ causet. Thus, as long as N is large enough to capture the relevant region of exponential expansion, increasing N further will have no effect on the results of interest.

- 如前文暗示，只要 $N \gg 1/p$ ，该场景下几乎所有物理规律都由 p 的选择决定。这一点从图 7 中可以很容易看出:(a) 较小 N 对应的数据点实际上是较大 N 对应数据点的子集；(b) $N = 1500$ 因果集的 $N_\diamond$ 最大 (最小) 值与 $N = 2500$ 因果集的对应值完全相同。因此，只要 N 足够大，足以覆盖指数膨胀的相关区域，进一步增大 N 对目标结果不会产生任何影响。

- As we will next describe, the results of fitting $N_\diamond^{\max}$ vs L datasets with $V_\diamond$ vs τ curves, the above mentioned points, in particular, mean that the dimension D which gives the best fit will only depend on p . On the other hand, if N is too small, then the causal set is not large enough to ”sample the region of interest,” and we will get poor results. This is manifested in Fig. 7 by the fact that the maximum $N_\diamond$ for $N = 500$ are substantially smaller than those for the larger causets.

- 接下来我们会说明，用 $V_\diamond$ - τ 曲线拟合 $N_\diamond^{\max}$ - L 数据集的结果，上述结论尤其意味着，给出最优拟合结果的维度 D 仅依赖于 p 。反之，如果 N 过小，因果集的大小不足以“采样目标区域”，我们就会得到较差的结果。这一点在图 7 中体现为: 较小因果集的 $N = 500$ 的最大 $N_\diamond$ 远小于更大因果集的对应值。

We now summarize the results of the above mentioned fitting. For details see [80].

我们现在总结上述拟合的结果，详见文献 [80]。

Results

结果

Figure 8 shows the $(N_\diamond^{\max}, L)$ pairs that typically arise in causets generated by originary percolation with a given p (and N). Each causet generated with a given p gives a unique set of $N_\diamond^{\max}$. Another causet with the same p generates another set that differs slightly from the ones before it. An average and a standard deviation is then calculated and used in the fit. Four sets of p and N are shown. Note that $N \gg p^{-1}$. As explained earlier, points with some of the smallest and some of the largest L values are excluded, and the rest is then fitted with a $(V_\diamond, \tau)$ curve for a $D+1$ -dimensional de Sitter spacetime, for several values of D . It is important to note that there is no a priori connection between p and D and consequently there is no preference for any value of D while fitting to begin with. The datasets, such as shown in Fig. 8, are then fitted for all values of D from 1 to 9 . There is a best-fit value and then the fit (visibly) deteriorates more and more as D is changed from the best-fit value of D in either direction. The results for several such exercises, for a variety of values of p , are summarized in Table 1 .

图 8 展示了在给定 p (以及 N) 的起源渗滤生成的因果集中通常出现的 $(N_\diamond^{\max}, L)$ 对。每个由给定 p 生成的因果集给出唯一的 $N_\diamond^{\max}$ 集合。另一个具有相同 p 的因果集生成的集合与之前的集合略有不同。随后计算平均值和标准差并用于拟合。图中展示了四组 p 与 N 。注意 $N \gg p^{-1}$ 。如前文所述，我们排除了部分 L 值最小和最大的点，随后对剩余点，针对多个 D 值，用 $D+1$ 维德西特时空的 $(V_\diamond, \tau)$ 曲线进行拟合。需要重点指出，p 和 D 之间没有先验关联，因此在拟合初始阶段，不会对任何 D 值有所偏好。对于类似图 8 所示的数据集，我们随后对 1 到 9 之间的所有 D 值都进行了拟合。存在一个最佳拟合值，当 D 偏离该最佳拟合值向任意方向变化时，拟合效果会 (明显地) 越来越差。针对多种不同 p 值开展的多次这类计算的结果汇总在表 1 中。

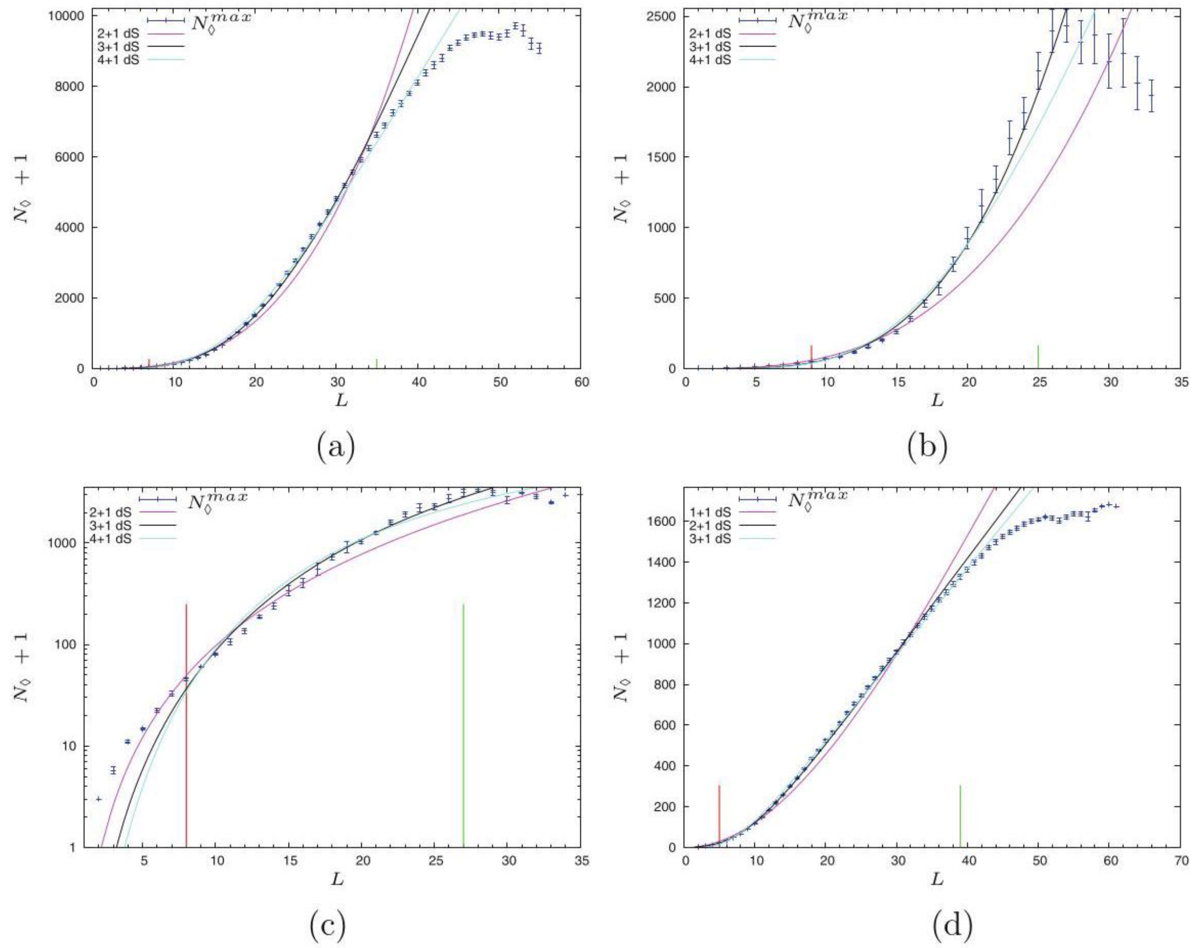


Fig. 8 $(N_\diamond^{\max}, L)$ pairs are plotted for four different causal sets generated by originary percolation with different values of p and N . They are the fitted with $(V_\diamond, \tau)$ curves for $D+1$ -dimensional de Sitter spacetime. The curves for the best-fit D (black) and for the two adjacent D values (pink and cyan) are shown. In figure (c) the data is represented and fitted on the log scale. (a) $N = 15000$ and $p = 0.001$. (b) $N = 50000$ and $p = 0.0001$. (c) $p = 0.0002$ and $N = 30000$. (d) $p = 0.01$ and $N = 2000$

图 8 绘制了四组不同的 $(N_{\diamond}^{\max}, L)$ 对，分别来自具有不同 p 和 N 值的起源渗滤生成的因果集。这些点随后用 $D+1$ 维德西特时空的 $(V_{\diamond}, \tau)$ 曲线拟合。图中展示了最佳拟合 D 对应的曲线 (黑色)，以及两个相邻 D 值对应的曲线 (粉色和青色)。子图 (c) 中的数据在对数刻度下展示和拟合。(a) $N = 15000$ 和 $p = 0.001$ 。(b) $N = 50000$ 和 $p = 0.0001$ 。(c) $p = 0.0002$ 和 $N = 30000$ 。(d) $p = 0.01$ 和 $N = 2000$

Table 1 Fitting parameters for some values of p .

表 1 若干 p 值对应的拟合参数。

p	n	$D+1$	l	m	χ	Fitting range in L	Number of runs
0.0001	50000	4	8.7 ± 1.8	2.105 ± 0.028	2.32	9-25	6
0.0002	30000	4	6.81 ± 0.72	1.926 ± 0.023	3.07	8-27	3
0.0005	20000	4	7.81 ± 0.57	1.787 ± 0.022	5.59	7-32	10
0.0008	15000	4	6.86 ± 0.22	1.749 ± 0.019	4.69	6-35	4
0.001	15000	4	6.20 ± 0.12	1.710 ± 0.013	4.97	7-35	23
0.003	15000	4	3.73 ± 0.013	1.483 ± 0.009	5.12	6–100	20
0.005	15000	4	3.097 ± 0.009	1.388 ± 0.009	4.69	5-150	20
0.01	2000	3	4.086 ± 0.028	1.136 ± 0.006	2.75	5-39	50
0.03	1000	3	2.331 ± 0.011	1.046 ± 0.006	0.663	5-53	5

Thus, the early universe generated by percolation undergoes this exponential expansion after already becoming spatially large during the random tree era. Random tree universe is very homogeneous to start with, but any leftover ”wrinkles” can be smoothed out by this de Sitter-like expansion. We will have more to say on this in the last section.

因此，由渗滤生成的早期宇宙在随机树时期已经成为空间大尺度结构后，会经历这种指数膨胀。随机树宇宙初始时十分均匀，但任何残留的“褶皱”都可以被这种类德西特膨胀抚平。我们将在最后一节对此展开更多讨论。

On a slightly different note, it is important to mention that a general question which naturally arises in causal set theory is whether causal sets which are well approximated by continua arise dynamically. It has been shown that the sequential growth models possess continuum limits, as $N \to \infty$ and $p \to 0$. However, the resulting continua look nothing like spacetime manifolds of dimension > 1 [84]. Nevertheless, the above discussion shows that it may still be the case that something resembling a spacetime may very well arise at finite p .

另一方面，需要指出，一个自然出现在因果集理论中的核心普遍问题是: 能被连续统良好近似的因果集是否能通过动力学过程产生。已经证明，顺序增长模型存在连续极限，如 $N \to \infty$ 和 $p \to 0$ 。然而，得到的连续统完全不同于维度为 > 1 的时空流形 [84]。尽管如此，上述讨论表明，在有限 p 下，类似时空的结构完全有可能产生。

Fluctuating Lambda

涨落的 Lambda

As mentioned earlier, the nature and magnitude of the dark energy are one of the most important problems of modern cosmology. The introduction of cosmological constant, which following [85] we will refer to as Λ , is a possible explanation for its nature but suffers from issues of naturalness when it comes to its magnitude. In this section, we will present a solution to this puzzle that relies heavily on the underlying discrete structure of causal set theory. It does assume the dark energy to be the cosmological constant, but with the difference that it has quantum fluctuations around a mean value that is set equal to zero. It can be shown that these fluctuations are of the right order to explain the observed magnitude of the dark energy. In this section, we will describe the main features of the argument that leads to the above conclusion.

如前所述，暗能量的本质与量级是现代宇宙学最重要的问题之一。引入宇宙常数 (沿用文献 [85] 的说法，我们将其记为 Λ) 是对暗能量本质的一种合理解释，但就其量级而言存在自然性问题。在本节中，我们将提出解决这一难题的方案，该方案高度依赖因果集理论的基础离散结构。该方案确实假设暗能量就是宇宙常数，但不同之处在于宇宙常数围绕平均值为零的均值存在量子涨落。可以证明，这些涨落的量级恰好可以解释观测到的暗能量量级。本节我们将介绍推导出上述结论的论证的核心特征。

Note that 4-volume and the cosmological constant are conjugate variables. In order to make this more transparent, thinks of GR as an effective theory, the gravitational action can be written as $S_G = \int \Lambda dV + \Lambda_1 \int R dV + \Lambda_2 \int R^2 dV + \ldots$. This expression makes it immediately clear that Λ and V are conjugate variables. Here R is the Ricci scalar [2]. Owing to this fact, it is a general expectation that in any theory of quantum gravity, the quantization will result in an uncertainty relationship between these two variables, i.e., a relationship of the kind $\Delta\Lambda\Delta V \sim 1$ (In natural units) is to be expected. This means that there ought to be quantum fluctuations in Λ around a mean value or $\Lambda = \langle\Lambda\rangle + \Delta\Lambda$. Of course, this expectation is valid only to the extent that these quantities and their fluctuations are defined in the particular QG theory we are talking about. For example, it is clearly borne out in unimodular gravity [86, 87] where the two variables have the status of fields except that the restriction to unimodularity renders ΔV meaningless making $\Delta\Lambda$ undefined. Do we expect this uncertainty relation to hold in causal set theory, which has a classical dynamics (i.e., CSG) but a quantum version is still lacking? Features of CSG, ideas about causal set quantization, and expectations about the continuum limit suggest that in all likelihood it should [74,85].

请注意，4-体积与宇宙常数是共轭变量。为了更清晰地说明这一点，我们将广义相对论视为一种有效理论，引力作用量可以写作 $S_G = \int \Lambda dV + \Lambda_1 \int R dV + \Lambda_2 \int R^2 dV + \ldots$ 。该表达式可以立即表明 Λ 和 V 是共轭变量。此处 R 是里奇标量 [2]。基于这一性质，我们普遍认为，在任何量子引力理论中，量子化都会导致这两个变量之间满足不确定性关系，即会存在 $\Delta\Lambda\Delta V \sim$ 形式的关系 (自然单位制下为 1)。这意味着 Λ 会围绕平均值或 $\Lambda = \langle\Lambda\rangle + \Delta\Lambda$ 产生量子涨落。当然，只有当这些量及其涨落是我们所讨论的特定量子引力理论中良定义的，这一推论才成立。例如，这一关系在幺模引力中得到了明确验证 [86, 87]，在该理论中这两个变量都具有场的性质，只不过对幺模性的约束使得 ΔV 失去意义，进而导致 $\Delta\Lambda$ 未定义。我们认为这一不确定性关系在因果集理论中也成立吗？因果集理论目前已有经典动力学 (即 CSG)，但其量子版本仍未完善。CSG 的性质、关于因果集量子化的思路，以及对连续极限的预期都表明，它极有可能满足这一关系 [74, 85]。

The correspondence between a causal set and the spacetime that approximates it is via the process of sprinkling, as has been mentioned before. This gives a definite meaning to the uncertainty in volume ΔV , i.e., $\Delta V \sim \pm\sqrt{N} \sim \pm\sqrt{V}$. This means that $\Delta\Lambda \sim 1/\Delta V \sim \pm 1/\sqrt{V}$, so that $\Lambda = \langle\Lambda\rangle \pm 1/\sqrt{V}$. If we now assume that $\langle\Lambda\rangle$ is zero for some reason, Λ still has a residual magnitude arising from quantum fluctuations. In standard cosmology, H^{-1} is the characteristic length and the radius of the observable universe is proportional to it [9]. This means that the 4-volume V is of the order of $\left(H^{-1}\right)^4$. This leads to $\Lambda = \pm 1/\sqrt{V} \sim \pm H^2 \sim \pm\rho_{\text{critical}}$, where ρ_{critical} is the total energy density (ρ_{total}) required to make the universe spatially flat [9]. Since we know that the universe is very close to being spatially flat, ρ_{critical} is almost the same as ρ_{total} and hence the magnitude of the residual fluctuations in Λ is of the same order as that of ρ_{total}, a fact corroborated by observations. As long as the universe remains close to being flat, which should be most or all of the history of the universe, this argument applies and the magnitude of Λ remains of the same order as the total energy density also solving the "Why Now?" puzzle mentioned in the latter half of the first section. It might be of interest here to mention that Work on spatially closed universe with $\rho_{\text{total}} > \rho_{\text{critical}}$ is in progress.

如前所述，因果集与近似它的时空之间的对应关系是通过撒播过程建立的。这赋予了体积不确定性 ΔV 明确的含义，即 $\Delta V \sim \pm\sqrt{N} \sim \pm\sqrt{V}$。这意味着 $\Delta\Lambda \sim 1/\Delta V \sim \pm 1/\sqrt{V}$，因此 $\Lambda = \langle\Lambda\rangle \pm 1/\sqrt{V}$。如果我们现在假设 $\langle\Lambda\rangle$ 因某种原因等于零，Λ 仍会存在由量子涨落产生的残余幅值。在标准宇宙学中，H^{-1} 是特征长度，可观测宇宙的半径与它成正比 [9]。这说明 4-体积 V 的量级为 $\left(H^{-1}\right)^4$。由此可得 $\Lambda = \pm 1/\sqrt{V} \sim \pm H^2 \sim \pm\rho_{\text{critical}}$，其中 ρ_{critical} 是使宇宙保持空间平坦所需的总能量密度 (ρ_{total}) [9]。我们已知宇宙非常接近空间平坦，因此 ρ_{critical} 几乎等于 ρ_{total}，故 Λ 残余涨落的幅值与 ρ_{total} 量级相同，这一结论得到了观测的证实。只要宇宙保持近似平坦——这应当符合宇宙大部分甚至全部历史的状态——该论证就成立，且 Λ 的幅值始终与总能量密度量级相同，这也解决了第一节后半部分提到的“为何是现在”难题。此处值得一提的是，针对带有 $\rho_{\text{total}} > \rho_{\text{critical}}$ 的闭合空间宇宙的研究正在进行中。

The next question that we need to ask is how this new Λ affects the evolution of the universe. This is very important as the standard model of cosmology has been able to accommodate/explain many important observations including the Big Bang nucleosynthesis (BBN), the existence of cosmic microwave background (CMB) radiation, and the age of the universe [88,89]. If the fluctuations in Λ adversely affect any of these observations, it would help us rule out the scenario (or put bounds on it). Similarly, we could look for a direct observation of these fluctuations. All this requires a knowledge of how Λ behaves as a function of time.

我们接下来需要探讨的问题是，这种新的 Λ 如何影响宇宙演化。这一点非常重要，因为标准宇宙学模型已经能够容纳/解释许多重要观测结果，包括大爆炸核合成 (BBN)、宇宙微波背景 (CMB) 辐射的存在以及宇宙年龄 [88,89]。如果 Λ 的涨落对上述任何观测结果产生不利影响，就能帮助我们排除这一情景 (或对它给出约束)。同理，我们也可以寻找这些涨落的直接观测证据。所有这些研究都需要了解 Λ 随时间的变化规律。

The Model

模型

Our original argument provides us with an order of magnitude estimate but does not tell us how to calculate Λ as a function of time. To this end, we would need a model that preserves all of the features of the original argument while letting us calculate Λ as a function of time. One such attempt at such a model [85] is summarized in the following.

我们最初的论证仅能给出数量级估计，无法告诉我们如何将 Λ 计算为时间的函数。为此，我们需要一个既能保留原论证所有特征、又允许我们计算 Λ 随时间变化的模型。下文将总结这类模型的一次尝试 [85]。

The gravitational action of a space without any ”excitations” (or in vacuum) is simply $S_g = \int \Lambda dV = \Lambda V$. This gives us a very meaningful interpretation of Λ as S_g/V , i.e., Λ is the action of free spacetime per unit volume. In terms of causal set theory, this translates to action of spacetime per spacetime atom (aka satom) or $\Lambda = S_g/N$. We again employ the picture that the universe ”comes into existence” with the ”birth” of the first element and then grows as elements come into being one at a time [74]. As each element is born, it contributes to the (gravitational) action. The most natural value of this contribution, call it α , is something of order unity. Let us say each element contributes exactly a unit of action, i.e., assume $\alpha = 1$. This means that when the universe is N elements old, the gravitational action $S_g = \sum \alpha_i = \sum 1 = N$ and $\Lambda = S_g/N = 1$, and we end up with the same magnitude problem. Now, let us further assume that these contributions randomly fluctuate in sign. In this case, the mean value of action at any stage vanishes but there are fluctuations about this mean value of zero. At stage N , i.e., after N elements are born, $S_g = \langle S\rangle + \Delta S = 0 \pm \sqrt{N}$, and thus $\Lambda = \pm\sqrt{N}/N = \pm 1/\sqrt{N} = \pm 1/\sqrt{V}$. Depending on how much stock one puts in this argument, it not only tells us the magnitude of the fluctuations, which happens to be in agreement with the original argument, but also explains why the mean value of Λ vanishes. We retain the nice feature that as the universe grows, Λ diminishes just enough to stay comparable to the total energy density of the universe with the added bonus that not only has a mechanism for the fluctuations been devised but we can now, in principle, create an ensemble of universes in each of which we have Λ as a function of time. All we have to do is flip a coin every time an element is born to determine the sign of its contribution and then add that contribution to the action. Dividing the accumulated action at any stage by the number of elements gives us the value of Λ at that stage. If we grow the universe to the currently observed size in this fashion, we have one member of the ensemble. Lastly, it should be noted that the sign of Λ , at least at this level of the argument, is uniformly random.

不存在任何“激发”的真空空间的引力作用量恰好是 $S_g = \int \Lambda dV = \Lambda V$ 。这为我们将 Λ 解读为 S_g/V 提供了非常合理的依据，即 Λ 是单位体积的自由时空作用量。在因果集合理论中，这对应每个时空原子 (也称 satom) 或 $\Lambda = S_g/N$ 的时空作用量。我们沿用这样的图景: 宇宙随第一个元素的“诞生”而“产生”，之后元素逐个生成，宇宙不断生长 [74]。每一个元素诞生时，都会对 (引力) 作用量产生贡献。这个贡献我们记为 α ，其自然取值是单位量级。我们不妨假设每个元素恰好贡献一个单位的作用量，即假定 $\alpha = 1$ 。这意味着当宇宙拥有 N 个元素时，引力作用量 $S_g = \sum \alpha_i = \sum 1 = N$ 且 $\Lambda = S_g/N = 1$ ，我们仍会面临相同的量级问题。现在我们进一步假设这些贡献的符号是随机涨落的。这种情况下，任意阶段作用量的平均值为零，但会存在围绕零均值的涨落。在阶段 N ，即 N 个元素诞生后，$S_g = \langle S\rangle + \Delta S = 0 \pm \sqrt{N}$ ，因此 $\Lambda = \pm\sqrt{N}/N = \pm 1/\sqrt{N} = \pm 1/\sqrt{V}$ 。根据对该论证的接受程度，它不仅给出了涨落的量级 (恰好与原论证一致)，还解释了为何 Λ 的均值为零。该模型保留了原论证的优良性质: 随着宇宙生长，Λ 的衰减幅度恰好能保持与宇宙总能量密度相当，额外的优势在于不仅给出了涨落的产生机制，原则上我们现在还可以生成宇宙系综，每个宇宙中都能得到 Λ 随时间的变化函数。我们只需在每个元素诞生时抛硬币确定其贡献的符号，再将该贡献加到总作用量中即可。在任意阶段将累积作用量除以元素数量，就能得到该阶段 Λ 的值。按照这种方式将宇宙生长到当前观测到的大小，我们就得到了系综中的一个成员。最后需要注意，至少在该论证层面，Λ 的符号是均匀随机的。

The Simulation

模拟

We are now, in principle, in a position to simulate the evolution of a universe that has fluctuating Λ as one of the components. Still, some practical issues need sorting before we can do that. The first minor one is the realization that there are around 10^{240} satoms in our observable universe. Adding such a large number of contributions one at a time is physically impossible. This means that we will have to take satoms in large chunks and use the central limit theorem to determine the sum of their contributions, of course, all the while making sure that the changes in quantities involved are small compared to their then current values.

原则上，我们现在已经可以对包含涨落 Λ 作为组分之一的宇宙演化进行模拟。但在开展模拟前，仍有若干实际问题需要解决。第一个小问题是: 我们可观测宇宙中约有 10^{240} 个时空原子，逐一添加这么多贡献在物理上是不可能的。这意味着我们必须将时空原子分成大块处理，利用中心极限定理计算总贡献，当然同时要始终保证相关物理量的变化量远小于它们当前的数值。

The most important issue that we still need to resolve is the interpretation of (or the choice of a measure for) V , the 4 -volume that appears in $\Lambda = \pm 1/\sqrt{N} = \pm 1/\sqrt{V}$ in a cosmological setting. If the observationally preferred "cosmological principle" is enforced (as mentioned in the first section), we have three choices for the geometry of the universe [2], and the total volume to the past of any hypersurface of homogeneity in two of them, i.e., the flat and the open cases, is infinite. A better choice, in such a scenario, is to use the volume related to the observable universe instead. Let us interpret V as the 4-volume of the past light cone. If we use the observationally favored spatially flat universe with the metric [9]

我们仍需解决的最重要问题是,如何解释 (或者说如何选择测度) 宇宙学场景中出现在 $\Lambda = \pm 1/\sqrt{N} = \pm 1/\sqrt{V}$ 里的 4-体积 V 。如果遵循观测支持的“宇宙学原理” (正如第一节所述)，宇宙的几何有三种可能 [2]；其中平直和开放这两种情况下，任意均匀超曲面过去的总体积都是无穷大。在这种情况下，更合适的选择是改用和可观测宇宙相关的体积。我们将 V 解释为过去光锥的 4-体积。如果采用观测上更偏向的、度规形式如下的空间平直宇宙 [9]

$$ds^2 = a(\eta)^2 \left[-d\eta^2 + dr^2 + r^2 d\Omega^2 \right], \tag{11}$$

the 4-volume of the past light cone is given by

过去光锥的 4-体积可表示为

$$V(\eta) = \frac{4\pi}{3} \int_0^\eta d\eta' a(\eta')^4 (\eta - \eta')^3. \tag{12}$$

We will use this as the definition of volume in our simulations. Here η is the conformal time, a is the scale factor, r is the radial coordinate, and, with polar angle θ and azimuthal angle $\phi, d\Omega^2 \equiv d\theta^2 + r^2 d\phi^2$ is the solid angle. Now, a step in the simulations will change the value of the (conformal) time by a small amount, which in turn can be used to calculate the small change in the scale factor (using the "equation of motion" to be discussed in the next paragraph). This change can then be used to update the volume and, consequently, to find out the number of satoms, ΔN , that produce this change in volume. We can then calculate the resulting change in the action as $\Delta S_i = \alpha \xi_i \sqrt{\Delta N_i}$ using the central limit theorem. .For completeness let us mention that this important result, which was presented by de Moivre in 1733, and which was later independently discovered by Gauss, says that the averages of the random samples of size N , which are derived from a distribution with mean μ and variance σ^2 , tend toward a normal distribution of the same mean with variance σ^2/N [90]. This is indeed a very deep result as nothing has been assumed about the distribution from which original samples are drawn. Here $\pm\alpha$ is the contribution of a satom to the action as it is born, and ξ is a random number drawn from a normal distribution with a vanishing mean and unit standard deviation. The subscript "i" stands for the iteration number or the stage in the simulation. The new Λ can now be calculated as

> 我们将在模拟中采用这个作为体积的定义。此处 η 是共形时间，a 是尺度因子，r 是径向坐标，结合极角 θ 和方位角 $\phi, d\Omega^2 \equiv d\theta^2 + r^2 d\phi^2$ 得到立体角。模拟中的每一步都会让 (共形) 时间产生一个微小增量，我们可以据此计算尺度因子的微小变化 (利用下一段将要讨论的“运动方程”)。得到这个变化后就可以更新体积，进而求出产生这次体积变化的时空原子数目 ΔN 。随后我们可以利用中心极限定理计算出作用量的对应变化为 $\Delta S_i = \alpha\xi_i\sqrt{\Delta N_i}$ 。为完整起见，这里需要说明: 该重要结论由棣莫弗在 1733 年提出，后被高斯独立发现；它指出，从均值为 μ 、方差为 σ^2 的分布中抽取大小为 N 的随机样本，样本均值会趋近于均值相同、方差为 σ^2/N 的正态分布 [90]。这确实是一个非常深刻的结论，因为它对原始样本的分布没有做任何假设。此处，$\pm\alpha$ 是一个时空原子诞生时对作用量的贡献，ξ 是从均值为零、单位标准差的正态分布中抽取的随机数。下标“i”代表迭代次数，即模拟的当前阶段。新的 Λ 现在可以计算为

$$\Lambda_{i+1} = \frac{S_{i+1}}{N_{i+1} l_f^4} \tag{13}$$

$$= \frac{S_i + \alpha\xi_{i+1}\sqrt{\Delta N_i}}{(N_i + \Delta N_i)\, l_f^4} \tag{14}$$

It is clear that this equation makes α a parameter of the model whose natural value is around 1 (in natural units).

> 显然这个方程说明 α 是模型的一个参数，其自然值约为 1(自然单位制下)。

Of course, we also need an "equation of motion" to update the scale-factor a itself. The two Friedman equations (in conformal coordinates) for the flat case are [2, 9]

> 当然，我们也需要一个“运动方程”来更新尺度因子 a 本身。平直情况下共形坐标下的两个弗里德曼方程为 [2, 9]

$$\frac{a'^2}{a^4} = \frac{1}{3}\left(\rho_{\text{matter}} + \rho_{\text{radiation}} + \Lambda\right), \tag{15}$$

and

> 以及

$$\frac{a''}{a^3} = \frac{1}{6}\left(\rho_{\text{matter}} + \rho_{\text{radiation}}\right) + \frac{2}{3}\Lambda - \frac{1}{2} p_{\text{radiation}} \,. \tag{16}$$

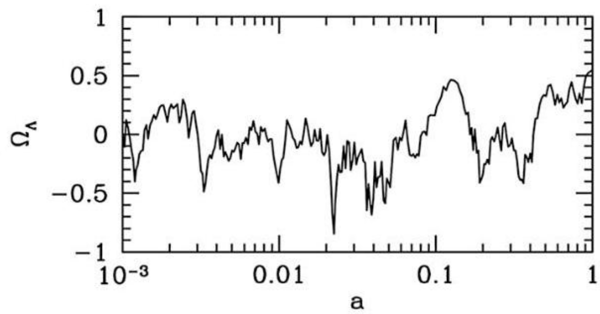
Here ”” denotes a derivative w.r.t. the conformal time, ρ is the energy density, and p is the pressure. We make Λ a function of time and use these equations as our dynamical guide. In this chapter, we will refer to the first one as the Hamiltonian constraint equation or HCEq and the second one as the acceleration equation or AccEq. Of course, making Λ a function of time leaves the two equations incompatible with each other and only one of them can be used. In fact, the AccEq does not render itself very well to numerical simulations when radiation is present in the mix. The instabilities in this case grow to dominate the true solution after a while [91]. Another choice is to make a linear combination of the two of the type $\mu\times$ AccEq $+\nu\times$ HCEq and use that to evolve the universe. Henceforth, we will refer to this as the MixedEq. This makes μ/υ the second parameter of the model. Note that the HCEq cannot be obtained by simply putting $\mu = 0$ in the MixedEq for it changes the order of the equation and is structurally different. Thus, we need to treat the HCEq separately. In conformal coordinates, the MixedEq is

此处”” 表示对共形时间的导数，ρ 是能量密度，p 是压强。我们令 Λ 为时间的函数，并将这些方程作为动力学指导。本章中，我们将第一个方程称为哈密顿约束方程 (HCEq)，第二个称为加速度方程 (AccEq)。当然，令 Λ 为时间函数会导致两个方程互不兼容，只能使用其中一个。实际上，当体系中存在辐射时，AccEq 并不适合数值模拟。这种情况下的不稳定性会在一段时间后增长并主导真实解 [91]。另一种选择是对两个方程做形如 $\mu\times$ AccEq $+\nu\times$ HCEq 的线性组合，用该组合描述宇宙演化。下文我们将其称为混合方程 (MixedEq)。由此，μ/υ 成为模型的第二个参数。请注意，HCEq 不能通过简单将 $\mu = 0$ 代入 MixedEq 得到，因为这会改变方程的阶数，二者结构不同。因此我们需要单独处理 HCEq。共形坐标下的 MixedEq 为

$$a'' = \frac{\mu - 3\upsilon}{2\mu}\frac{a'^2}{a} + \frac{\upsilon a^3}{2\mu}\rho_{\text{total}} - \frac{a^3 p_{\text{total}}}{2}, \tag{17}$$

Fig. 9 The fraction of energy in Λ , i.e., $\Omega_\Lambda = \rho_\Lambda/\rho_{\text{total}}$ plotted against the scale factor. The figure roughly covers the period between the current epoch and the surface of last scattering [9]. Ω_Λ fluctuates as a result of fluctuations in $\rho_\Lambda\,(=\Lambda)$

图 9 Λ 中的能量占比，即 $\Omega_\Lambda = \rho_\Lambda/\rho_{\text{total}}$ ，随标度因子变化的曲线。该图大致覆盖了当前纪元到最后散射面之间的时段 [9]。Ω_Λ 因 $\rho_\Lambda\,(=\Lambda)$ 的涨落而发生涨落



with

其中

$$\rho_{\text{total}} = \rho_{\text{radiation}} + \rho_{\text{matter}} + \Lambda, \tag{18}$$

and

且

$$p_{\text{total}} = \frac{\rho_{\text{radiation}}}{3} - \Lambda. \tag{19}$$

As mentioned earlier, the AccEq is numerically unstable when radiation is the dominant energy component, and hence it is no surprise that as we increase μ , we encounter the same instability. In fact, a simple analysis shows [91] that all stable values of μ/υ lie within the interval (0-3]. We keep ourselves in this interval while describing the results of the simulations in the next section.

如前文所述，当辐射是主导能量成分时，AccEq 数值不稳定，因此不难理解，随着我们增大 μ ，会遇到同样的不稳定性。实际上，简单分析即可证明 [91]，μ/υ 的所有稳定值都落在区间 (0, 3] 内。在下一节介绍模拟结果时，我们将取值保持在该区间内。

The Results

结果

The first result worth mentioning is that the model is insensitive to the parameter μ/υ and essentially every feature of the model is only dictated by α [91]. This means that all dynamical equations generate almost the same behavior, which in turn shows that the model is structurally stable and the results do not depend on the choice of a special dynamical equation. Here we want to discuss some of the main features of the simulation results.

第一个值得一提的结果是，该模型对参数 μ/υ 不敏感，模型的几乎所有特性都仅由 α 决定 [91]。这意味着所有动力学方程都产生几乎相同的行为，进而表明该模型结构稳定，结果不依赖于特定动力学方程的选择。下面我们来讨论模拟结果的几个主要特性。

1. The model produces fluctuations in Λ , both in magnitude and in sign. Figure 9 depicts a typical run that shows these fluctuations, which are important if we still want to account for observations like the Big Bang nucleosynthesis that can be easily destroyed if we have a ”large” (positive or negative) Λ most of the time. Fluctuations mean that Λ remains ”small” for an appreciable amount of time and the probability that it will not change BBN and other important results is satisfactorily high [92].

1. 模型会产生 Λ 的涨落，涨落同时体现在大小和符号上。图 9 展示了一次典型运行中的这类涨落——如果我们还需要解释大爆炸核合成这类观测，这些涨落就十分重要: 如果大部分时间里 Λ 都处于“较大”(正值或负值) 状态，大爆炸核合成就很容易被破坏。涨落意味着 Λ 会在相当长的时间内保持“较小”，因此它不改变 BBN 及其他重要结论的概率足够高 [92]。

2. In standard cosmology (i.e., with constant Λ), any of the equations that we are using can be used as a dynamical guide, and they all, of course, produce the same unique solution for the time dependence of the scale factor for a given set of initial conditions. This is obviously not true anymore. Not only are there fluctuations in the magnitude of Λ , which get translated into fluctuations in the rate of change of the scale factor, but there are fluctuations in sign, which have a very important effect. In the HCEq, i.e., $3H^2 = \rho + \Lambda$

, it simply means that when we have a large negative fluctuation such that the RHS of the HCEq becomes negative, we cannot proceed as an imaginary H has not (yet) been assigned any meaningful interpretation [85]. In the MixedEq, however, we do not have that problem. A large negative fluctuation can still make the total energy negative but that only affects the rate of change of H and not H itself. This can eventually make H change its sign, that is, an expansion can become a contraction, but numerically and conceptually, there is no problem. However, if a contraction persists, it starts to increase matter and radiation densities, whereas at the same time, fluctuations in Λ keep on diminishing in magnitude because the past 4- volume is still increasing, and after a while, Λ becomes too small to reverse that contraction. So in both cases (i.e., with the HCEq and the MixedEq), we do not reach the present observable size of the universe. On the other hand, this makes acceleration (large positive Λ), deceleration (nondominant negative Λ), and contraction (large and dominant negative Λ) natural phases of the evolution of the universe. Depending on how much importance one assigns to this model, this is very satisfying, at least philosophically, as it assigns no special meaning to the current phase of accelerated expansion. However, it is still true that contrary to the standard cosmology, not all of the simulations with this fluctuating Λ reach the present size of the universe (Or equivalently the present value of the temperature, i.e., around 3 K).

2. 在标准宇宙学中 (即 Λ 为常数时)，我们使用的任意一个方程都可以作为动力学指导，当然，在给定初始条件下，它们都给出尺度因子时间演化的同一个唯一解。这种情况在我们的模型中显然不再成立。不仅 Λ 的大小存在涨落 (涨落会转化为尺度因子变化率的涨落)，其符号也存在涨落，后者影响十分显著。在 HCEq 即 $3H^2 = \rho + \Lambda$ 中，当出现大的负涨落使得 HCEq 的右侧变为负数时，我们就无法继续计算，因为虚数 H (目前) 还没有赋予任何有物理意义的解释 [85]。但在 MixedEq 中不存在这个问题: 大的负涨落仍会使总能量变为负数，但这只会影响 H 的变化率，不会影响 H 本身。这最终可能使 H 改变符号，即膨胀转为收缩，但无论在数值上还是概念上都不存在问题。不过，如果收缩持续下去，物质和辐射密度会不断升高，与此同时，Λ 涨落的幅度会持续减小，因为过去 4-体积仍在增加，一段时间后 Λ 会变得过小，无法逆转收缩。因此在两种情况 (即采用 HCEq 和 MixedEq) 下，我们都无法得到宇宙现在的可观测大小。另一方面，这也使得加速 (大的正 Λ)、减速 (非主导的负 Λ) 和收缩 (大且主导的负 Λ) 成为宇宙演化的自然阶段。不管人们认为这个模型的重要性如何，至少在哲学层面这一点十分令人满意，因为它没有给当前的加速膨胀阶段赋予特殊意义。但事实仍然是: 与标准宇宙学不同，不是所有涨落 Λ 的模拟都能得到宇宙现在的大小 (或者等效地说，现在的温度，即大约 3 K)。

This percentage of the universes that do not ”make it” depends on α . ”Larger” values of this parameter naturally mean larger fluctuations happen more often, increasing the probability of a contraction in case of the MixedEq or a ”collapse of the scheme” in case of the HCEq. Figure 10 shows the ratio of the successful runs as a function of μ/v . For a relatively ”smaller” value of $\alpha = 1/50$, almost all of the simulations reach the current size. Figure 11 shows the same for a larger value of $\alpha = 1/30$. The reason we do not have such a graph for $\alpha \sim 1$ is that as we approach $\alpha = 1$ from below, which should be its natural value, almost none of the simulations reach the present size. It seems like a mild fine-tuning problem in the case of the HCEq, although the MixedEq solves the issue to a large extent.

没能“成功演化到现在”的宇宙比例取决于 α 。该参数取值“越大”，大涨落自然就越频繁，对于 MixedEq 会提高收缩发生的概率，对于 HCEq 则会提高“框架坍缩”的概率。图 10 展示了成功运行的比例随 μ/υ 的变化关系: 当 $\alpha = 1/50$ 取值相对“较小”时，几乎所有模拟都能演化到当前宇宙大小。图 11 是 $\alpha = 1/30$ 取值较大时的相同结果。我们没有给出 $\alpha \sim 1$ 的对应图像，原因是当我们从下方趋近 $\alpha = 1$ ($\alpha = 1$ 是它的自然取值) 时，几乎没有模拟能演化到当前宇宙大小。对于 HCEq 来说，这似乎是一个轻微的精细调节问题，不过 MixedEq 在很大程度上解决了这个问题。

3. The question whether or not the model generates fluctuations that can explain the observed magnitude of the dark energy is very important. Since ours is a probabilistic model, we can only assign probabilities. Figure 12 shows histograms of the final values of the fraction of the energy density carried by Λ , denoted by $\Omega_\Lambda^{\text{final}}$ (Ω has been defined earlier and accordingly $\Omega_\Lambda = \frac{\Lambda}{\Lambda + \rho_{\text{ radiation}} + \rho_{\text{ matter}}}$). The observations indicate that today around 70 percent of the energy density is carried by the dark energy. In other words, the observationally favored value of $\Omega_\Lambda^{\text{final}}$ is around 0.7 . It turns out that we obtain $\Omega_\Lambda^{\text{final}} > 0.5$ in about 5 percent of the simulations. $\Omega_\Lambda^{\text{final}} \geq 0.7$ is, of course, rarer and happens only around 2 percent of the time. Obviously, these probabilities depend on the value of α increasing which increases this probability considerably, but on the flip side, we increase the probability of ”prematurely” contracting the universe. So there is a tension in the model between getting higher probabilities for $\Omega_\Lambda^{\text{final}}$ and reaching the present size of the universe.

3. 该模型能否产生可解释观测到的暗能量幅度的涨落，这一问题十分重要。由于我们的模型是概率模型，我们仅能分配概率。图 12 展示了由 Λ 携带的能量密度占比最终值的直方图，该占比记为 $\Omega_\Lambda^{\text{final}}$ (Ω ($\Omega_\Lambda^{\text{final}}$ (Ω 已在之前定义，因此有 $\Omega_\Lambda = \frac{\Lambda}{\Lambda + \rho_{\text{ radiation}} + \rho_{\text{ matter}}}$)。观测表明，如今约 70% 的能量密度由暗能量携带。换言之，观测支持的 $\Omega_\Lambda^{\text{final}}$ 取值约为 0.7。结果显示，约 5% 的模拟中得到了 $\Omega_\Lambda^{\text{final}} > 0.5$ 。当然，$\Omega_\Lambda^{\text{final}} \geq 0.7$ 更为罕见，仅在约 2% 的情况中发生。显然，这些概率依赖于 α 的取值，α 增大时该概率会显著提升，但另一方面，宇宙“过早”收缩的概率也会随之上升。因此该模型中，获得更高的 $\Omega_\Lambda^{\text{final}}$ 概率与达到宇宙当前尺寸之间存在一种制衡。

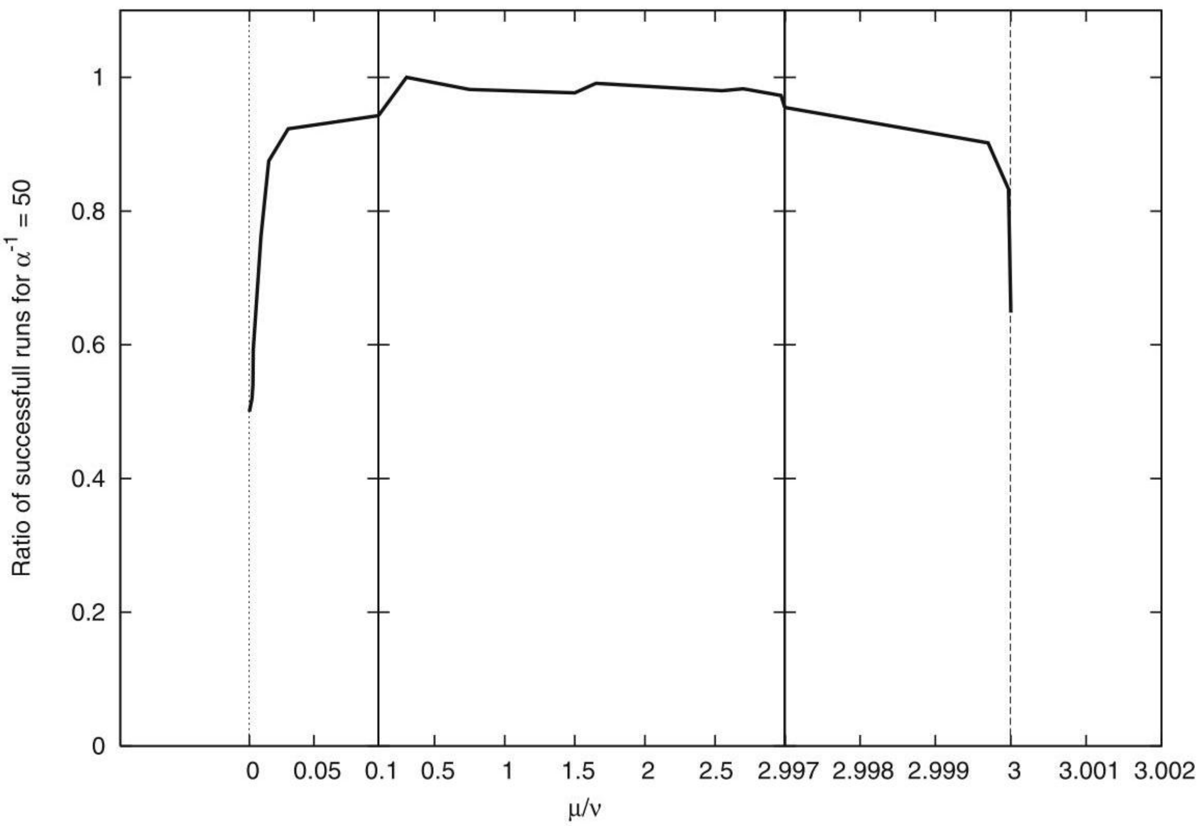


Fig. 10 Shows the percentage of the runs that reach the present value of the temperature for different values of the parameter μ/υ and for $\alpha = 1/50$. Near the end points of the stable interval [0-3], the horizontal scale is blown up as the change is quite steep. For the bulk of the interval, the transition between different μ/υ values looks smooth

图 10 展示了不同参数 μ/υ 取值下，以及对应 $\alpha = 1/50$ 时，达到当前温度值的模拟运行占比。在稳定区间 [0-3] 的端点附近，由于变化十分陡峭，横轴做了放大处理。在区间的大部分区域，不同 μ/υ 取值之间的过渡看起来是平滑的

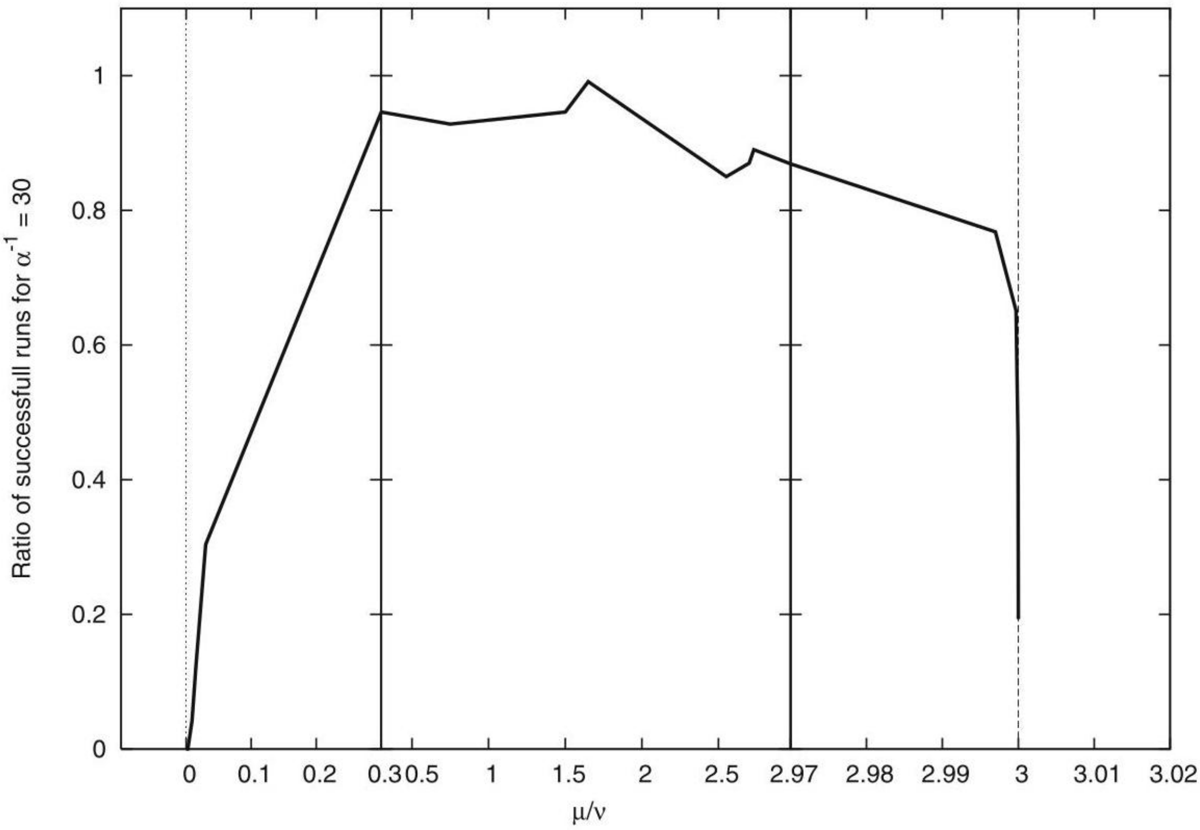


Fig. 11 Same as Fig. 10 but for $\alpha^{-1} = 30$

图 11 与图 10 相同，但针对 $\alpha^{-1} = 30$

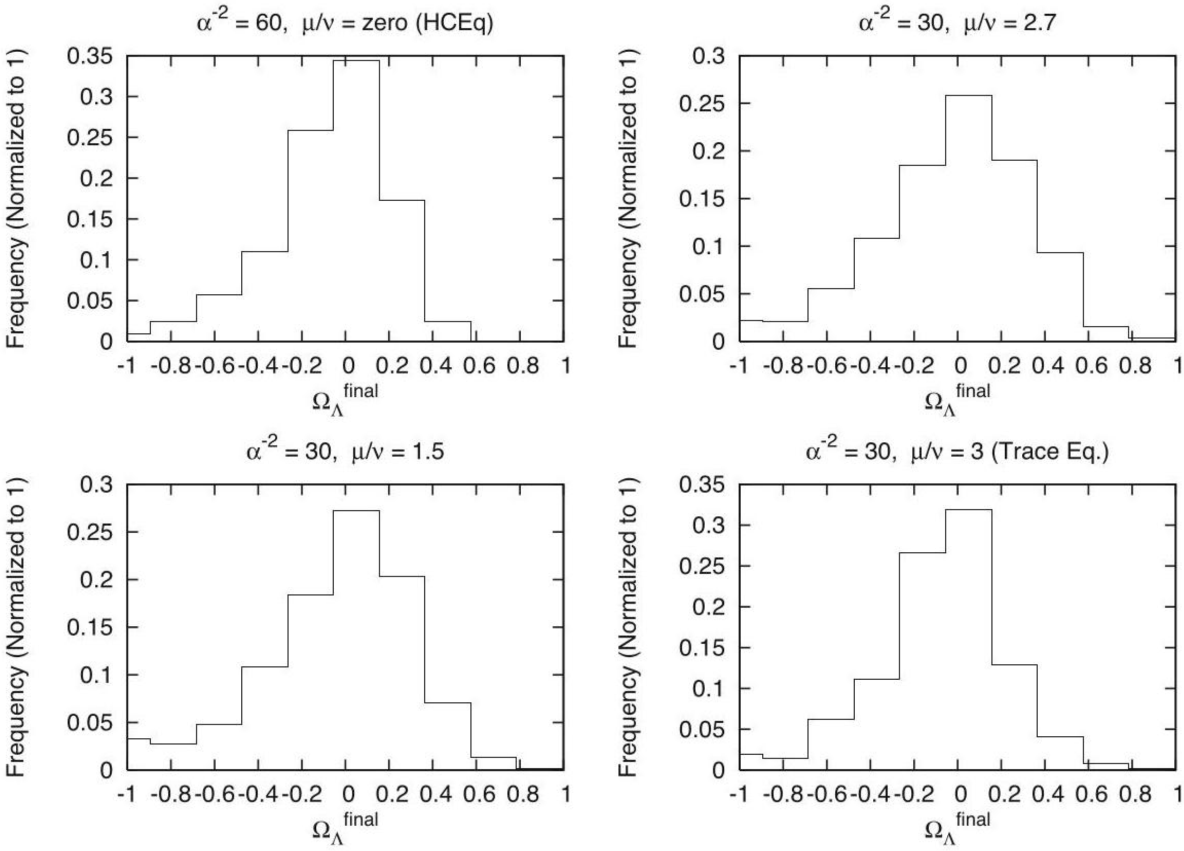


Fig. 12 Final values of the fraction of the energy density carried by Λ for different values of α . If Λ is not constant in time, then the value of Ω_Λ quoted in the literature is an integrated effect and not just the final value of the quantity but $\Omega_\Lambda^{\mathrm{final}}$ should correlate well with it especially for small values of redshift

图 12 不同 α 取值下，由 Λ 携带的能量密度占比的最终值。若 Λ 不随时间恒定，文献中给出的 Ω_Λ 取值是积分效应，并非仅该物理量的最终值，但 $\Omega_\Lambda^{\mathrm{final}}$ 应当与该积分效应有良好相关性，尤其在红移较小的情况下

4. As mentioned in the first section, we expect Λ to stay comparable to the total energy density a significant amount of the time as long as the universe is expanding. This expectation is again confirmed by the model that shows a tracking behavior. Figure 13 shows the (absolute magnitude of the) energy density in Λ as a function of the scale factor. Initially, radiation is dominant and Λ tracks radiation. Later on matter overtakes and Λ stops tracking radiation and starts tracking matter. This is extremely satisfying as it shows that the current epoch is not special but is one of a natural cycle of acceleration, deceleration, and contraction.

4. 如第一节所述，我们预期只要宇宙处于膨胀中，Λ 在相当长的时间内都会保持与总能量密度相当。该预期再次得到模型验证，模型显示了追踪行为。图 13 展示了 Λ 中能量密度 (绝对值) 随标度因子的变化。初始阶段辐射主导，Λ 追踪辐射。后续物质取代辐射成为主导，Λ 停止追踪辐射，开始追踪物质。这一结果非常令人满意，它表明当前宇宙时期并非特殊时期，而是加速、减速、收缩自然循环中的一环。

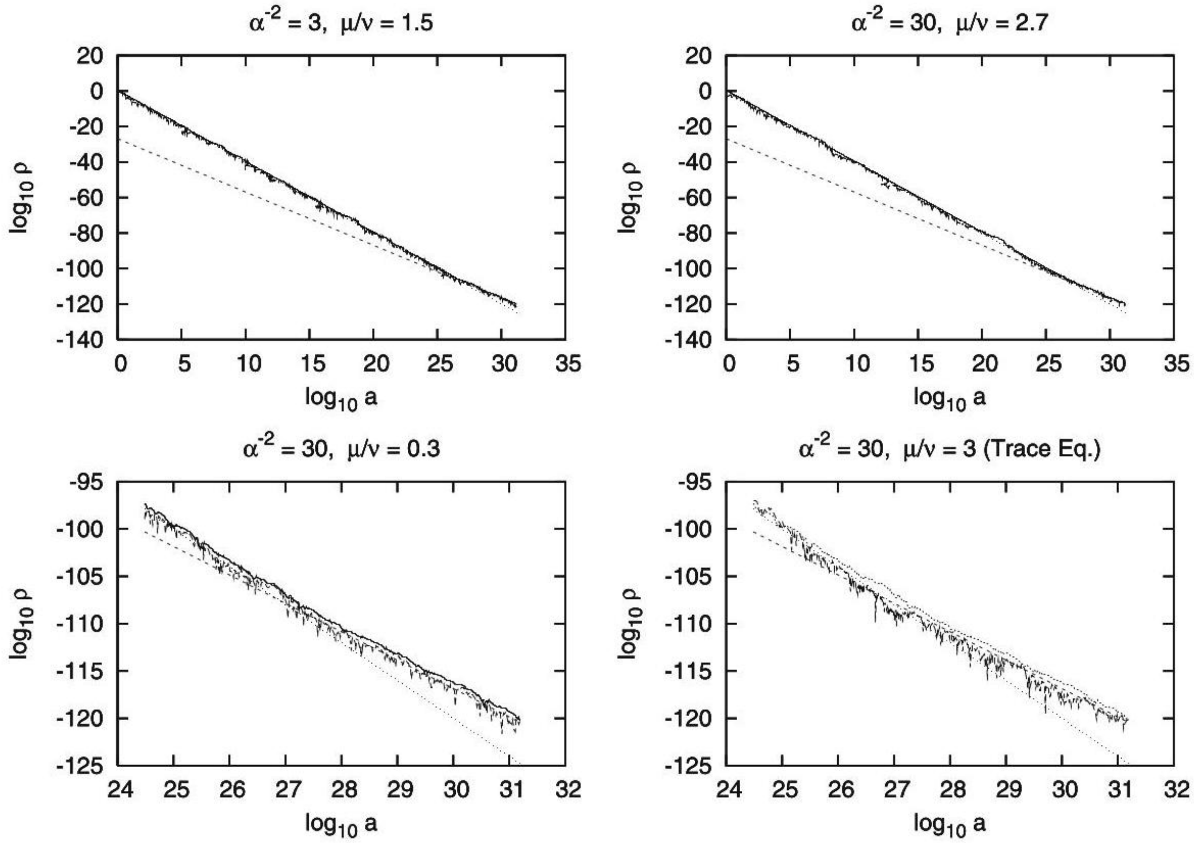


Fig. 13 Typical tracking behavior shown by the magnitude of Λ . Top two figures show this behavior starting with the Planck time until the present epoch, whereas the bottom two figures focus on the transition from a^{-4} scaling (radiation domination) shown by the dotted line to a^{-3} scaling (matter domination) shown by the dashed line of the ambient or total energy density. It clearly shows that the energy density in Λ (solid jittery line) switches its scaling as well and follows the total energy density modulo fluctuations shown by wiggly dotted line

图 13 Λ 幅度展现的典型追踪行为。上方两幅图展示了从普朗克时间到当前纪元的该行为，下方两幅图则聚焦于从环境总能量密度的 a^{-4} 标度 (辐射主导，由虚线表示) 到 a^{-3} 标度 (物质主导，由短划线表示) 的转变。图中清楚显示，Λ 中的能量密度 (抖动实线) 也会改变自身标度，在涨落 (由波浪虚线展示) 的基础上跟随总能量密度

5. There is an interesting trend in the simulations that shows that once we get a ”dominant” Λ that naturally results in a large (close to 1) Ω_Λ , it has a tendency to stay approximately constant over appreciable redshifts (unpublished work). Figure 14 shows this trend for some simulations where Ω_Λ becomes large at late times and then remains large for an appreciable amount of time, sometimes for several redshifts.

5. 模拟中出现了一个有趣的趋势: 一旦得到“占主导”的 Λ，自然会产生接近 1 的大 Ω_Λ，且它有在相当大的红移范围内保持近似恒定的趋势 (未发表成果)。图 14 展示了部分模拟中的这一趋势，这些模拟中 Ω_Λ 在晚期变大，之后在很长一段时间内 (有时跨越数个红移) 都维持大取值。

In summary, causal set theory motivates a tracking model [85] that is structurally stable [91], matches the observations [92] with reasonable probabilities, and makes acceleration, deceleration, and contraction natural phases of the evolution of the universe.

总而言之，因果集合理论推导出了一个结构稳定的追踪模型 [85][91]，该模型以合理的概率符合观测结果 [92]，并使得加速、减速和收缩成为宇宙演化的自然阶段。

Summary and Concluding Remarks

总结与结论

In the absence of direct data to test their predictions against, theories of quantum gravity could use cosmology as their testing ground. Causal sets are a very simple and clearly defined attempt at quantizing gravity that assumes a discrete structure for our universe at the most fundamental level. Fortunately, it has reached a stage in its development where it can say something about some of the important puzzles present in standard cosmology. Here is what we have discussed in this chapter about causal set cosmology.

在缺乏直接数据验证其预测的情况下，量子引力理论可以将宇宙学作为其试验场。因果集是量子化引力的一种非常简洁清晰的方案，它假设我们的宇宙在最基础层面是离散结构。幸运的是，该理论的发展已经进入了可以解答标准宇宙学中若干重要谜题的阶段。本章我们对因果集宇宙学的讨论内容如下。

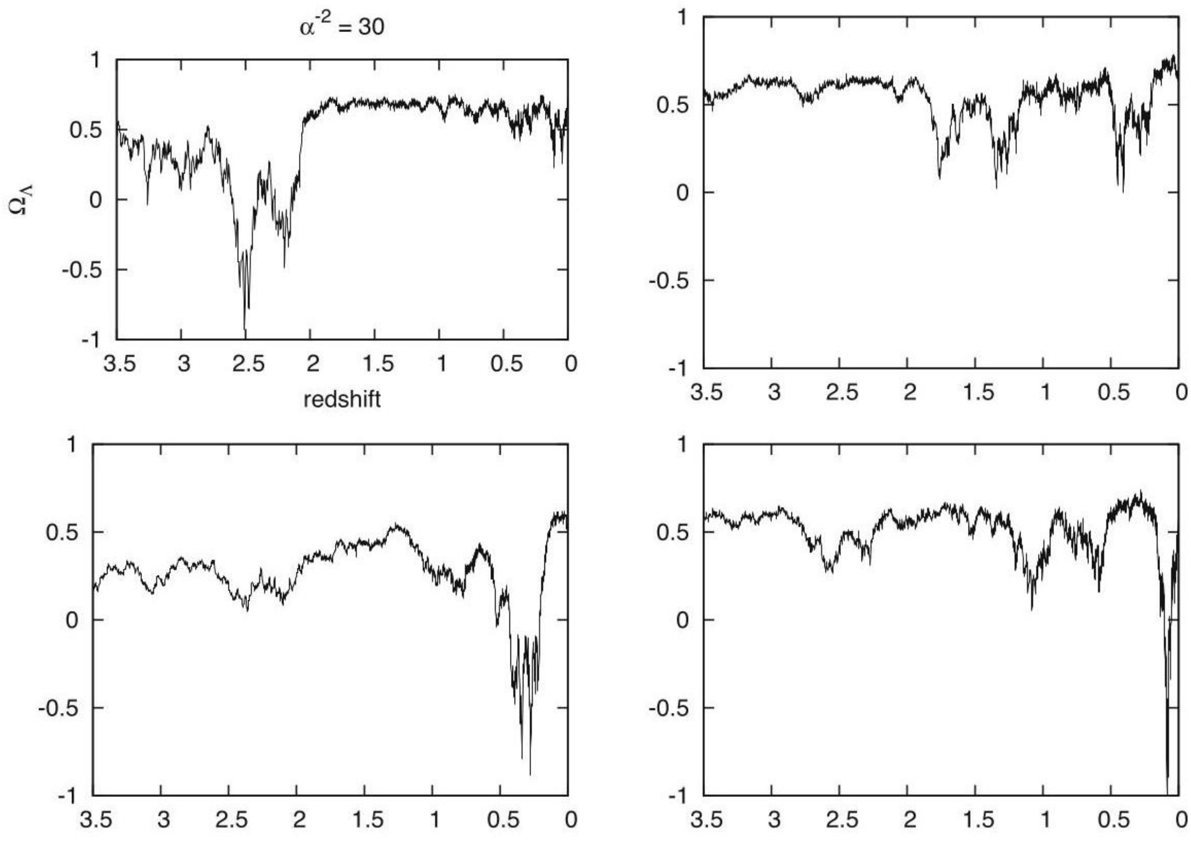


Fig. 14 Plots of Ω_Λ vs redshift for some simulations that show that once Ω_Λ becomes large, it tends to stay that way

图 14 部分模拟得到的 Ω_Λ 随红变化的关系图，结果表明一旦 Ω_Λ 变大，它就会倾向于维持该状态

- The standard model of cosmology does not tell us where the universe comes from. In fact, if the theory of general relativity is supposed to be valid all the way to time $t = 0$, the universe ends up in a singularity, where not only do the physical laws not apply, but it is impossible to get any information from $t < 0$. Thus, it is impossible to know what happens "before" the singularity. On the other hand, if causal set cosmology is taken seriously, one still has a "beginning" or a Big Bang in the model, but the singularity is not a problem anymore. The post, which acts as the initial singularity, is like any other element in the theory. In other words, discreteness can "resolve" the Big Bang singularity. In fact, the same post is the big crunch singularity of the previous cycle of the universe. One can in principle calculate the probability of post occurrence in any of the CSG models, and it is easy to see what happens to the universe before and after the post is formed.

- 标准宇宙学模型无法告诉我们宇宙的起源。实际上，如果广义相对论从始至终都适用于时间 $t = 0$ ，那么宇宙会终结于奇点——在奇点不仅物理定律失效，而且无法从 $t < 0$ 获得任何信息。因此我们不可能知道奇点“之前”发生了什么。另一方面，如果采信因果集宇宙学，模型中仍然存在“开端”，也就是大爆炸，但奇点不再是问题。作为初始奇点的“桩点”和该理论中的其他元素没有区别。换句话说，离散性可以“消解”大爆炸奇点。实际上，这个桩点就是宇宙上一个循环中大挤压的奇点。我们原则上可以在任意 CSG 模型中计算桩点出现的概率，也能够很清晰地得到桩点形成前后宇宙的演化情况。

- Every time the universe collapses (to a post) and then bounces back, the effective behavior of the expansion can be described as if the whole causal set started with that post with renormalized coupling constants. Since the percolation dynamics is an attractive fixed point under this renormalization flow in the space of CSG models that have posts, one may start the universe generically in any of these models, and it eventually will end up arbitrarily close to percolation. This makes percolation the natural candidate for the study of the posts and also guarantees the results are free of any kind of fine-tuning in the space of models.

- 每当宇宙坍缩成一个桩点后再反弹，膨胀的有效行为就可以描述为: 整个因果集从该桩点开始，耦合常数已经过重整化。由于在含桩点的 CSG 模型空间中，渗滤动力学是该重整化流下的吸引不动点，因此无论宇宙初始属于这类模型中的哪一个，最终都会任意趋近于渗滤态。这让渗滤成为研究桩点的自然选择，也保证了结果不依赖模型空间中的任何精细调节。

- The early universe generated by percolation has two clearly separable eras. The first of these resembles a random tree, where the spatial volume of the universe increases exponentially with the "cosmological time." As the universe accumulates $1/p$ elements after the post, where p is the parameter of the percolation, it enters a de Sitter-like phase.

- 渗滤产生的早期宇宙有两个清晰可分的时期。早期宇宙类似随机树，宇宙的空间体积随“宇宙学时间”指数增长。当桩点之后宇宙积累了 $1/p$ 个元素——其中 p 是渗滤的参数——宇宙就进入类德西特相。

- One of the most unsettling problems of the standard cosmology is the fact that the universe appears very homogeneous on large scales - something that can be seen directly in the cosmic microwave background temperature isotropy. The percolation universe as it emerges from its early phase is very homogeneous in the sense that any neighborhood looks like any other. Every element has the same sort of past and future and the same sort of "neighborhoods." Thus, the model has a very strong potential for solving the homogeneity problem as it naturally favors a homogeneity in the initial conditions. This is particularly true if the matter is generated by the structure in the causal set itself. On the other hand, if we put external degrees of freedom on the causal set, it may happen that even if we start with different initial conditions for these degrees of freedom, the de Sitter-like expansion gets rid of this inhomogeneity. Of course, there are random fluctuations that cause deviations away from homogeneity. These fluctuations might prove helpful in solving another extremely important puzzle in the early universe, namely, the origin of density perturbations that seed the late time structure formation.

- 标准宇宙学中最令人不安的问题之一是: 宇宙在大尺度上看起来高度均匀——这一点可以从宇宙微波背景温度的各向同性直接观测到。从早期阶段演化而来的渗滤宇宙非常均匀，任意区域都和其他区域性质一致。每个元素的过去、未来和“邻域”性质都相同。因此，该模型天然倾向于初始条件的均匀性，非常有潜力解决均匀性问题。当物质本身由因果集结构生成时，这一点尤为明显。另一方面，如果我们在因果集上引入外部自由度，那么即使这些自由度初始条件不同，类德西特膨胀也会消除这种不均匀性。当然，随机涨落会导致对均匀性的偏离，这些涨落反而可能有助于解决早期宇宙中另一个极为重要的谜题，也就是为后期结构形成提供种子的密度扰动的起源问题。

- Another puzzle is the large size of the universe compared to, say, the Planck length, when the universe is still very young, say, something like 100 Planck times old. This is related to both the horizon problem and the flatness puzzle. Models with percolation dynamics naturally generate a large size of the universe. If we start a percolation model with parameter p , the spatial volume becomes of the order of p^{-1} within $\ln p^{-1}$ time steps. Depending on how small p is, the universe can be made arbitrarily large. Since cosmic renormalization provides a mechanism which can drive the effective value of p to arbitrarily small values if one waits long enough, there is no fine-tuning involved.

- 另一个谜题是: 当宇宙还十分年轻，比如仅有约 100 个普朗克时间年龄时，它的尺寸相对于普朗克长度就已经非常大了。这一问题与视界问题和平坦性谜题都相关。渗滤动力学模型天然就能产生大尺寸宇宙。如果我们从参数为 p 的渗滤模型出发，空间体积会在 $\ln p^{-1}$ 个时间步内达到 p^{-1} 的量级。根据 p 的取值大小，可以构造出任意大尺寸的宇宙。由于宇宙重整化提供了一种机制，只要等待足够久，就能将 p 的有效值驱动到任意小的值，因此该过程不涉及精细调节。

- Drawing on ideas from causal set theory and assuming an uncertainty relationship between the cosmological term, Λ , and the 4-volume, it can be shown that the cosmological "constant" need not be a fixed parameter. Rather, it fluctuates about zero with a magnitude proportional to $1/\sqrt{V}, V$ being some measure of the past four volume. The amplitude of these fluctuations is then of the right order of magnitude to explain the dark energy in the universe.

- 基于因果集理论的观点，假设宇宙学常数 Λ 与 4-体积之间存在不确定关系，可以证明宇宙学“常数”不必是固定参数。它会在零附近涨落，涨落幅度与 $1/\sqrt{V}, V$ 成正比，$1/\sqrt{V}, V$ 是过去四维体积的某种度量。这类涨落的振幅量级恰好可以解释宇宙中的暗能量。

- The abovementioned argument about the magnitude of Λ is so general that it would apply at all times, and, indeed, we expect the energy density in the cosmological ”constant” to always be of order the ambient density in the universe.

- 上述关于 Λ 量级的论证十分普适，适用于所有时期，事实上我们也预期宇宙学“常数”的能量密度始终处于宇宙环境密度的量级。

- The model as we have demonstrated is largely free of any fine-tuning. Also there is nothing special about the present epoch in this model. If dark energy is explained as these fluctuations in Λ , then it not only solves the ”Why Now?” problem but also makes accelerated or decelerated expansions, and even contractions, regular phases in the evolution of the universe. Thus, it makes the assertion that there is nothing special about the present phase of accelerated expansion.

- 正如我们所展示的，该模型基本不存在精细调谐问题，且当前纪元在该模型中也并无特殊之处。如果暗能量可以用 Λ 的这类涨落解释，那么它不仅解决了“为何现在出现”问题，还将加速膨胀、减速膨胀甚至收缩都变成了宇宙演化的常规阶段。因此该模型主张，当前的加速膨胀阶段并无特殊之处。

The most attractive feature, one might say, of the solutions to the puzzles of cosmology presented in this chapter is the fact that they are conceptually (radically) different from most other solutions. Some of these ideas are readily testable. For example, it is a practical certainty (unpublished work) that if Λ is fluctuating, it must have changed sign between now and some time in the past, say, a redshift of 3 . This can be checked by some of the currently running observational programs. Also important for its testability is the notion that it may have affected the evolution of the universe at early times. For example, it may have affected the Big Bang nucleosynthesis. In [92] it has shown that it not only can pass the constraints coming from BBN, among others, but can actually help ease some of the tensions present in standard cosmology, e.g., on account of BBN and the age of the universe. Also, the primordial generation of density perturbations can be achieved without resorting to a scalar field, which can be produced at the level of the spacetime: the universe that emerges from the random tree era is highly homogeneous but has small random fluctuations in its structure. Similarly, fluctuations in Λ have the potential to seed these fluctuations as well. Last, but not the least, is the potential to provide an alternative scenario to compete against the currently popular ones - an important tool in the advancement of scientific knowledge.

可以说，本章提出的这些宇宙学谜题解决方案最吸引人的特点，在于它们在概念上 (彻底) 不同于绝大多数其他方案。其中部分观点是可直接检验的。例如，一个已有实践依据的结论 (未发表工作) 指出: 若 Λ 确实存在涨落，那么它一定会在现在和过去某个时刻 (比如红移为 3 时) 之间改变符号，这一点可以通过目前正在运行的多个观测项目验证。该理论可检验性的另一个要点在于，它认为 Λ 的涨落可能影响了宇宙早期演化，例如可能影响了大爆炸核合成。文献 [92] 已经表明，该理论不仅满足大爆炸核合成 (BBN) 等给出的约束，实际上还能帮助缓解标准宇宙学中存在的一些矛盾，例如 BBN 与宇宙年龄之间的矛盾。此外，原初密度扰动的产生无需引入标量场，扰动可以在时空层面生成: 从随机树时代演化而来的宇宙具有高度均匀性，同时结构上存在微小随机涨落。类似地，Λ 的涨落也有能力成为这些扰动的种子。最后但同样重要的是，该理论有可能提供一种替代方案，与目前流行的理论竞争——这是推动科学知识进步的重要途径。

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